THEORY

OF

EXISTENCE:

PART I.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE MOTIONS THAT RESULT FROM THE COLLISION OF PONDERABLE BODIES.

BY

ELIAS DEXTER.

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DEFINITIONS AND EXPLANATIONS.

I.

The VELOCITY of a body is determined by multiplying the weight of the body by the space passed over in a given time.

II.

The MOMENTUM of a body is determined by multiplying the weight by the velocity of the body.

III.

While enunciating the laws which determine the results of collision in those cases in which two bodies only are concerned, the letters A and B will be taken to represent the bodies; and when three bodies are concerned, the third body will be represented by the letter C: that is, C will represent the body which is supposed to act upon B in conjunction with A.

IV.

The given momentum of the bodies before collision is all that will be accounted for after collision. Hence, no account will be taken of any resistance which the bodies may meet with from any source external to themselves. And therefore the assigned results are not to be considered as the actual, but as the theoretical results of the collision of the bodies: that is, they are the results that would be produced—

(1.) If no other than the given forces acted upon the bodies.

(2.) If A and C, at the time of their collision with B, were moving in lines of direction that would pass through the centre of gravity of B.

(3.) If the bodies were spherical, and retained the same shape and weight during and after, as before, collision.

V.

With respect to the comparative weights of A and B, three cases will be considered; namely:

(1.) When the weight of A is less than the weight of B.

(2.) When the weight of A is equal to the weight of B.

(3.) When the weight of A is greater than the weight of B.

With respect to the comparative weight of A and C with respect to B, two cases will be considered; namely, when the joint weight of A and C is equal to, or less than, the weight of B; and when the joint weight of A and C is equal to, or greater than, the weight of B.

VI.

With respect to the state or condition of B, at the time of collision, there are five cases that will be considered; namely:

(1.) When B is in a state of rest with respect to A.

(2.) When B is moving in the same right line and direction as that in which A is moving.

(3.) When B is moving in the same right line as that in which A is moving, but in an opposite direction.

(4.) When B is moving in a line of direction that will form any angle whatever with that in which A is moving.

(5.) When B is at rest with respect to A and C, while A and C move in lines of direction that will form any angle whatever.

SECTION I.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE FIRST CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS LESS THAN, OR EQUAL TO, THE WEIGHT OF B; AND IN WHICH B IS AT REST AT THE TIME OF COLLISION.

IN every case belonging to this class, the results of collision will be determined by the following laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. Both the velocity and momentum of A,—as compared with the velocity and momentum of A before collision,—will be proportional to the difference between the respective weights of A and B.

LAW 2. The momentum of B,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B.

LAW 3. The velocity of B,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B.

These laws will give the results of collision in terms of the weight of B: that is, if B, in any given case, should weigh ten pounds, the results of collision would all come out in *tenths*. If B should weigh nine pounds, the results would all come out in *ninths*. If B should weigh eight pounds, the results would all come out in *eighths;* and so on, for all weights whatsoever.

And the results, in each and every case, may all be

expressed by three fractional numbers: namely, that which expresses the difference between the respective weights of A and B; that which expresses the ratio of the weight of A to the weight of B; and that which expresses the square of the ratio of the weight of A to the weight of B.

The following tables have been drawn up for the purpose of showing the results of collision in four hundred different cases belonging to the first class. These tables must be read from left to right, across the page; for the problem and the solution thereof, are both given on the same line. The problems are all of one kind, and may be thus stated: Given, the respective weights of A and B, and the velocity and momentum of A before collision: to find the velocity and momentum of both A and B after collision. As no account is taken of any resistance which the bodies may meet with, either before or after collision, from any external source, the amount of the momentum to be accounted for after collision, will always be equal to the momentum assigned to A before collision. And since all the momentum before collision belongs to A; and since A, by supposition, comes in collision with B only; the whole of the momentum assigned to A before collision, must be found in A and B after collision. It is required to determine what portion of the momentum assigned to A, before collision, will be imparted to B at the time of collision; and what portion will be retained by A after it has come in collision with B.

In case No. 90, Table IV., of this section, for example, the weight of A is 10 pounds, and the velocity of A, before collision, is 10 feet per second. One of these numbers multiplied by the other will produce 100. Hence, the momentum of A, before collision, is put down

in the table as 100; and this is the amount of the momentum that is to be accounted for after collision. The weight of B is sixteen pounds, and B is supposed to be at rest at the time of collision.

Now, according to Law 1, both the velocity and momentum of A after collision,—as compared with the velocity and momentum of A before collision,—will be proportional to the difference between the respective weights of A and B. In case No. 90, Table IV., the difference between the weights of A and B is $\frac{6}{16}$. Hence, both the velocity and momentum of A, after collision, is put down in the table as $\frac{6}{16}$: the meaning of which is, that the velocity of A will be equal to six-sixteenths of the velocity which A had before collision: that is, equal to six-sixteenths of ten feet per second; and that the momentum of A will be equal to six-sixteenths of the momentum which A had before collision: that is, equal to six-sixteenths of 100.

According to Law 2, the momentum of B,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B. In case No. 90, that ratio is $\frac{10}{16}$. Hence, the momentum of B, after collision, is put down in the table as $\frac{10}{16}$; and the meaning of it is, that it will be equal to ten-sixteenths of the momentum which A had before collision: that is, equal to ten-sixteenths of 100; which, added to six-sixteenths,—the momentum of A,—will make sixteen-sixteenths of 100; which is equal to the momentum of A before collision.

According to Law 3, the velocity of B,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B. As that ratio, in case No. 90, is $\frac{10}{16}$, the velocity of B, after collision, is put down in the table as

$\frac{10}{16}^2$: that is, $\frac{10}{16} \times \frac{10}{16} = \frac{100}{256}$. And the meaning of it is, that it will be equal to one hundred two hundred and fifty-sixths of the velocity which A had before collision: that is, equal to $\frac{100}{256}$ of 10 feet per second.

In the following tables the results after collision are always compared with the given velocity and momentum of A before collision, as in the foregoing example.

Now, whatever may be the weight of A, or the weight of B, or whatever may be the velocity of A before collision; the Laws above enunciated will hold good in every case in which the weight of A is less than, or equal to, the weight of B, and in which B is at rest, with respect to A, at the time of collision.

TABLE I.

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	$\frac{1}{100}$	$\frac{100}{100}$	10	$\frac{10}{100}$		$\frac{99}{100}$	$\frac{99}{100}$	$\frac{1}{100}$	$\frac{1^2}{100}$
2	$\frac{2}{100}$	$\frac{100}{100}$	10	$\frac{20}{100}$		$\frac{98}{100}$	$\frac{98}{100}$	$\frac{2}{100}$	$\frac{2^2}{100}$
3	$\frac{3}{100}$	$\frac{100}{100}$	10	$\frac{30}{100}$		$\frac{97}{100}$	$\frac{97}{100}$	$\frac{3}{100}$	$\frac{3^2}{100}$
4	$\frac{4}{100}$	$\frac{100}{100}$	10	$\frac{40}{100}$		$\frac{96}{100}$	$\frac{96}{100}$	$\frac{4}{100}$	$\frac{4^2}{100}$
5	$\frac{5}{100}$	$\frac{100}{100}$	10	$\frac{50}{100}$		$\frac{95}{100}$	$\frac{95}{100}$	$\frac{5}{100}$	$\frac{5^2}{100}$
6	$\frac{6}{100}$	$\frac{100}{100}$	10	$\frac{60}{100}$		$\frac{94}{100}$	$\frac{94}{100}$	$\frac{6}{100}$	$\frac{6^2}{100}$
7	$\frac{7}{100}$	$\frac{100}{100}$	10	$\frac{70}{100}$		$\frac{93}{100}$	$\frac{93}{100}$	$\frac{7}{100}$	$\frac{7^2}{100}$
8	$\frac{8}{100}$	$\frac{100}{100}$	10	$\frac{80}{100}$		$\frac{92}{100}$	$\frac{92}{100}$	$\frac{8}{100}$	$\frac{8^2}{100}$
9	$\frac{9}{100}$	$\frac{100}{100}$	10	$\frac{90}{100}$		$\frac{91}{100}$	$\frac{91}{100}$	$\frac{9}{100}$	$\frac{9^2}{100}$
10	$\frac{10}{100}$	$\frac{100}{100}$	10	$\frac{100}{100}$		$\frac{90}{100}$	$\frac{90}{100}$	$\frac{10}{100}$	$\frac{10^2}{100}$
11	$\frac{11}{100}$	$\frac{100}{100}$	10	$\frac{110}{100}$		$\frac{89}{100}$	$\frac{89}{100}$	$\frac{11}{100}$	$\frac{11^2}{100}$
12	$\frac{12}{100}$	$\frac{100}{100}$	10	$\frac{120}{100}$		$\frac{88}{100}$	$\frac{88}{100}$	$\frac{12}{100}$	$\frac{12^2}{100}$
13	$\frac{13}{100}$	$\frac{100}{100}$	10	$\frac{130}{100}$		$\frac{87}{100}$	$\frac{87}{100}$	$\frac{13}{100}$	$\frac{13^2}{100}$
14	$\frac{14}{100}$	$\frac{100}{100}$	10	$\frac{140}{100}$		$\frac{86}{100}$	$\frac{86}{100}$	$\frac{14}{100}$	$\frac{14^2}{100}$
15	$\frac{15}{100}$	$\frac{100}{100}$	10	$\frac{150}{100}$		$\frac{85}{100}$	$\frac{85}{100}$	$\frac{15}{100}$	$\frac{15^2}{100}$
16	$\frac{16}{100}$	$\frac{100}{100}$	10	$\frac{160}{100}$		$\frac{84}{100}$	$\frac{84}{100}$	$\frac{16}{100}$	$\frac{16^2}{100}$
17	$\frac{17}{100}$	$\frac{100}{100}$	10	$\frac{170}{100}$		$\frac{83}{100}$	$\frac{83}{100}$	$\frac{17}{100}$	$\frac{17^2}{100}$
18	$\frac{18}{100}$	$\frac{100}{100}$	10	$\frac{180}{100}$		$\frac{82}{100}$	$\frac{82}{100}$	$\frac{18}{100}$	$\frac{18^2}{100}$
19	$\frac{19}{100}$	$\frac{100}{100}$	10	$\frac{190}{100}$		$\frac{81}{100}$	$\frac{81}{100}$	$\frac{19}{100}$	$\frac{19^2}{100}$
20	$\frac{20}{100}$	$\frac{100}{100}$	10	$\frac{200}{100}$		$\frac{80}{100}$	$\frac{80}{100}$	$\frac{20}{100}$	$\frac{20^2}{100}$
21	$\frac{21}{100}$	$\frac{100}{100}$	10	$\frac{210}{100}$		$\frac{79}{100}$	$\frac{79}{100}$	$\frac{21}{100}$	$\frac{21^2}{100}$
22	$\frac{22}{100}$	$\frac{100}{100}$	10	$\frac{220}{100}$		$\frac{78}{100}$	$\frac{78}{100}$	$\frac{22}{100}$	$\frac{22^2}{100}$
23	$\frac{23}{100}$	$\frac{100}{100}$	10	$\frac{230}{100}$		$\frac{77}{100}$	$\frac{77}{100}$	$\frac{23}{100}$	$\frac{23^2}{100}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	$\frac{24}{100}$	$\frac{100}{100}$	10	$\frac{240}{100}$		$\frac{76}{100}$	$\frac{76}{100}$	$\frac{24}{100}$	$\frac{24^2}{100}$
25	$\frac{25}{100}$	$\frac{100}{100}$	10	$\frac{250}{100}$		$\frac{75}{100}$	$\frac{75}{100}$	$\frac{25}{100}$	$\frac{25^2}{100}$
26	$\frac{26}{100}$	$\frac{100}{100}$	10	$\frac{260}{100}$		$\frac{74}{100}$	$\frac{74}{100}$	$\frac{26}{100}$	$\frac{26^2}{100}$
27	$\frac{27}{100}$	$\frac{100}{100}$	10	$\frac{270}{100}$		$\frac{73}{100}$	$\frac{73}{100}$	$\frac{27}{100}$	$\frac{27^2}{100}$
28	$\frac{28}{100}$	$\frac{100}{100}$	10	$\frac{280}{100}$		$\frac{72}{100}$	$\frac{72}{100}$	$\frac{28}{100}$	$\frac{28^2}{100}$
29	$\frac{29}{100}$	$\frac{100}{100}$	10	$\frac{290}{100}$		$\frac{71}{100}$	$\frac{71}{100}$	$\frac{29}{100}$	$\frac{29^2}{100}$
30	$\frac{30}{100}$	$\frac{100}{100}$	10	$\frac{300}{100}$		$\frac{70}{100}$	$\frac{70}{100}$	$\frac{30}{100}$	$\frac{30^2}{100}$
31	$\frac{31}{100}$	$\frac{100}{100}$	10	$\frac{310}{100}$		$\frac{69}{100}$	$\frac{69}{100}$	$\frac{31}{100}$	$\frac{31^2}{100}$
32	$\frac{32}{100}$	$\frac{100}{100}$	10	$\frac{320}{100}$		$\frac{68}{100}$	$\frac{68}{100}$	$\frac{32}{100}$	$\frac{32^2}{100}$
33	$\frac{33}{100}$	$\frac{100}{100}$	10	$\frac{330}{100}$		$\frac{67}{100}$	$\frac{67}{100}$	$\frac{33}{100}$	$\frac{33^2}{100}$
34	$\frac{34}{100}$	$\frac{100}{100}$	10	$\frac{340}{100}$		$\frac{66}{100}$	$\frac{66}{100}$	$\frac{34}{100}$	$\frac{34^2}{100}$
35	$\frac{35}{100}$	$\frac{100}{100}$	10	$\frac{350}{100}$		$\frac{65}{100}$	$\frac{65}{100}$	$\frac{35}{100}$	$\frac{35^2}{100}$
36	$\frac{36}{100}$	$\frac{100}{100}$	10	$\frac{360}{100}$		$\frac{64}{100}$	$\frac{64}{100}$	$\frac{36}{100}$	$\frac{36^2}{100}$
37	$\frac{37}{100}$	$\frac{100}{100}$	10	$\frac{370}{100}$		$\frac{63}{100}$	$\frac{63}{100}$	$\frac{37}{100}$	$\frac{37^2}{100}$
38	$\frac{38}{100}$	$\frac{100}{100}$	10	$\frac{380}{100}$		$\frac{62}{100}$	$\frac{62}{100}$	$\frac{38}{100}$	$\frac{38^2}{100}$
39	$\frac{39}{100}$	$\frac{100}{100}$	10	$\frac{390}{100}$		$\frac{61}{100}$	$\frac{61}{100}$	$\frac{39}{100}$	$\frac{39^2}{100}$
40	$\frac{40}{100}$	$\frac{100}{100}$	10	$\frac{400}{100}$		$\frac{60}{100}$	$\frac{60}{100}$	$\frac{40}{100}$	$\frac{40^2}{100}$
41	$\frac{41}{100}$	$\frac{100}{100}$	10	$\frac{410}{100}$		$\frac{59}{100}$	$\frac{59}{100}$	$\frac{41}{100}$	$\frac{41^2}{100}$
42	$\frac{42}{100}$	$\frac{100}{100}$	10	$\frac{420}{100}$		$\frac{58}{100}$	$\frac{58}{100}$	$\frac{42}{100}$	$\frac{42^2}{100}$
43	$\frac{43}{100}$	$\frac{100}{100}$	10	$\frac{430}{100}$		$\frac{57}{100}$	$\frac{57}{100}$	$\frac{43}{100}$	$\frac{43^2}{100}$
44	$\frac{44}{100}$	$\frac{100}{100}$	10	$\frac{440}{100}$		$\frac{56}{100}$	$\frac{56}{100}$	$\frac{44}{100}$	$\frac{44^2}{100}$
45	$\frac{45}{100}$	$\frac{100}{100}$	10	$\frac{450}{100}$		$\frac{55}{100}$	$\frac{55}{100}$	$\frac{45}{100}$	$\frac{45^2}{100}$
46	$\frac{46}{100}$	$\frac{100}{100}$	10	$\frac{460}{100}$		$\frac{54}{100}$	$\frac{54}{100}$	$\frac{46}{100}$	$\frac{46^2}{100}$
47	$\frac{47}{100}$	$\frac{100}{100}$	10	$\frac{470}{100}$		$\frac{53}{100}$	$\frac{53}{100}$	$\frac{47}{100}$	$\frac{47^2}{100}$
48	$\frac{48}{100}$	$\frac{100}{100}$	10	$\frac{480}{100}$		$\frac{52}{100}$	$\frac{52}{100}$	$\frac{48}{100}$	$\frac{48^2}{100}$
49	$\frac{49}{100}$	$\frac{100}{100}$	10	$\frac{490}{100}$		$\frac{51}{100}$	$\frac{51}{100}$	$\frac{49}{100}$	$\frac{49^2}{100}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	$\frac{50}{100}$	$\frac{100}{100}$	10	$\frac{500}{100}$		$\frac{50}{100}$	$\frac{50}{100}$	$\frac{50}{100}$	$\frac{50^2}{100}$
51	$\frac{51}{100}$	$\frac{100}{100}$	10	$\frac{510}{100}$		$\frac{49}{100}$	$\frac{49}{100}$	$\frac{51}{100}$	$\frac{51^2}{100}$
52	$\frac{52}{100}$	$\frac{100}{100}$	10	$\frac{520}{100}$		$\frac{48}{100}$	$\frac{48}{100}$	$\frac{52}{100}$	$\frac{52^2}{100}$
53	$\frac{53}{100}$	$\frac{100}{100}$	10	$\frac{530}{100}$		$\frac{47}{100}$	$\frac{47}{100}$	$\frac{53}{100}$	$\frac{53^2}{100}$
54	$\frac{54}{100}$	$\frac{100}{100}$	10	$\frac{540}{100}$		$\frac{46}{100}$	$\frac{46}{100}$	$\frac{54}{100}$	$\frac{54^2}{100}$
55	$\frac{55}{100}$	$\frac{100}{100}$	10	$\frac{550}{100}$		$\frac{45}{100}$	$\frac{45}{100}$	$\frac{55}{100}$	$\frac{55^2}{100}$
56	$\frac{56}{100}$	$\frac{100}{100}$	10	$\frac{560}{100}$		$\frac{44}{100}$	$\frac{44}{100}$	$\frac{56}{100}$	$\frac{56^2}{100}$
57	$\frac{57}{100}$	$\frac{100}{100}$	10	$\frac{570}{100}$		$\frac{43}{100}$	$\frac{43}{100}$	$\frac{57}{100}$	$\frac{57^2}{100}$
58	$\frac{58}{100}$	$\frac{100}{100}$	10	$\frac{580}{100}$		$\frac{42}{100}$	$\frac{42}{100}$	$\frac{58}{100}$	$\frac{58^2}{100}$
59	$\frac{59}{100}$	$\frac{100}{100}$	10	$\frac{590}{100}$		$\frac{41}{100}$	$\frac{41}{100}$	$\frac{59}{100}$	$\frac{59^2}{100}$
60	$\frac{60}{100}$	$\frac{100}{100}$	10	$\frac{600}{100}$		$\frac{40}{100}$	$\frac{40}{100}$	$\frac{60}{100}$	$\frac{60^2}{100}$
61	$\frac{61}{100}$	$\frac{100}{100}$	10	$\frac{610}{100}$		$\frac{39}{100}$	$\frac{39}{100}$	$\frac{61}{100}$	$\frac{61^2}{100}$
62	$\frac{62}{100}$	$\frac{100}{100}$	10	$\frac{620}{100}$		$\frac{38}{100}$	$\frac{38}{100}$	$\frac{62}{100}$	$\frac{62^2}{100}$
63	$\frac{63}{100}$	$\frac{100}{100}$	10	$\frac{630}{100}$		$\frac{37}{100}$	$\frac{37}{100}$	$\frac{63}{100}$	$\frac{63^2}{100}$
64	$\frac{64}{100}$	$\frac{100}{100}$	10	$\frac{640}{100}$		$\frac{36}{100}$	$\frac{36}{100}$	$\frac{64}{100}$	$\frac{64^2}{100}$
65	$\frac{65}{100}$	$\frac{100}{100}$	10	$\frac{650}{100}$		$\frac{35}{100}$	$\frac{35}{100}$	$\frac{65}{100}$	$\frac{65^2}{100}$
66	$\frac{66}{100}$	$\frac{100}{100}$	10	$\frac{660}{100}$		$\frac{34}{100}$	$\frac{34}{100}$	$\frac{66}{100}$	$\frac{66^2}{100}$
67	$\frac{67}{100}$	$\frac{100}{100}$	10	$\frac{670}{100}$		$\frac{33}{100}$	$\frac{33}{100}$	$\frac{67}{100}$	$\frac{67^2}{100}$
68	$\frac{68}{100}$	$\frac{100}{100}$	10	$\frac{680}{100}$		$\frac{32}{100}$	$\frac{32}{100}$	$\frac{68}{100}$	$\frac{68^2}{100}$
69	$\frac{69}{100}$	$\frac{100}{100}$	10	$\frac{690}{100}$		$\frac{31}{100}$	$\frac{31}{100}$	$\frac{69}{100}$	$\frac{69^2}{100}$
70	$\frac{70}{100}$	$\frac{100}{100}$	10	$\frac{700}{100}$		$\frac{30}{100}$	$\frac{30}{100}$	$\frac{70}{100}$	$\frac{70^2}{100}$
71	$\frac{71}{100}$	$\frac{100}{100}$	10	$\frac{710}{100}$		$\frac{29}{100}$	$\frac{29}{100}$	$\frac{71}{100}$	$\frac{71^2}{100}$
72	$\frac{72}{100}$	$\frac{100}{100}$	10	$\frac{720}{100}$		$\frac{28}{100}$	$\frac{28}{100}$	$\frac{72}{100}$	$\frac{72^2}{100}$
73	$\frac{73}{100}$	$\frac{100}{100}$	10	$\frac{730}{100}$		$\frac{27}{100}$	$\frac{27}{100}$	$\frac{73}{100}$	$\frac{73^2}{100}$
74	$\frac{74}{100}$	$\frac{100}{100}$	10	$\frac{740}{100}$		$\frac{26}{100}$	$\frac{26}{100}$	$\frac{74}{100}$	$\frac{74^2}{100}$
75	$\frac{75}{100}$	$\frac{100}{100}$	10	$\frac{750}{100}$		$\frac{25}{100}$	$\frac{25}{100}$	$\frac{75}{100}$	$\frac{75^2}{100}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	$\frac{76}{100}$	$\frac{100}{100}$	10	$\frac{760}{100}$		$\frac{24}{100}$	$\frac{24}{100}$	$\frac{76}{100}$	$\frac{76^2}{100}$
77	$\frac{77}{100}$	$\frac{100}{100}$	10	$\frac{770}{100}$		$\frac{23}{100}$	$\frac{23}{100}$	$\frac{77}{100}$	$\frac{77^2}{100}$
78	$\frac{78}{100}$	$\frac{100}{100}$	10	$\frac{780}{100}$		$\frac{22}{100}$	$\frac{22}{100}$	$\frac{78}{100}$	$\frac{78^2}{100}$
79	$\frac{79}{100}$	$\frac{100}{100}$	10	$\frac{790}{100}$		$\frac{21}{100}$	$\frac{21}{100}$	$\frac{79}{100}$	$\frac{79^2}{100}$
80	$\frac{80}{100}$	$\frac{100}{100}$	10	$\frac{800}{100}$		$\frac{20}{100}$	$\frac{20}{100}$	$\frac{80}{100}$	$\frac{80^2}{100}$
81	$\frac{81}{100}$	$\frac{100}{100}$	10	$\frac{810}{100}$		$\frac{19}{100}$	$\frac{19}{100}$	$\frac{81}{100}$	$\frac{81^2}{100}$
82	$\frac{82}{100}$	$\frac{100}{100}$	10	$\frac{820}{100}$		$\frac{18}{100}$	$\frac{18}{100}$	$\frac{82}{100}$	$\frac{82^2}{100}$
83	$\frac{83}{100}$	$\frac{100}{100}$	10	$\frac{830}{100}$		$\frac{17}{100}$	$\frac{17}{100}$	$\frac{83}{100}$	$\frac{83^2}{100}$
84	$\frac{84}{100}$	$\frac{100}{100}$	10	$\frac{840}{100}$		$\frac{16}{100}$	$\frac{16}{100}$	$\frac{84}{100}$	$\frac{84^2}{100}$
85	$\frac{85}{100}$	$\frac{100}{100}$	10	$\frac{850}{100}$		$\frac{15}{100}$	$\frac{15}{100}$	$\frac{85}{100}$	$\frac{85^2}{100}$
86	$\frac{86}{100}$	$\frac{100}{100}$	10	$\frac{860}{100}$		$\frac{14}{100}$	$\frac{14}{100}$	$\frac{86}{100}$	$\frac{86^2}{100}$
87	$\frac{87}{100}$	$\frac{100}{100}$	10	$\frac{870}{100}$		$\frac{13}{100}$	$\frac{13}{100}$	$\frac{87}{100}$	$\frac{87^2}{100}$
88	$\frac{88}{100}$	$\frac{100}{100}$	10	$\frac{880}{100}$		$\frac{12}{100}$	$\frac{12}{100}$	$\frac{88}{100}$	$\frac{88^2}{100}$
89	$\frac{89}{100}$	$\frac{100}{100}$	10	$\frac{890}{100}$		$\frac{11}{100}$	$\frac{11}{100}$	$\frac{89}{100}$	$\frac{89^2}{100}$
90	$\frac{90}{100}$	$\frac{100}{100}$	10	$\frac{900}{100}$		$\frac{10}{100}$	$\frac{10}{100}$	$\frac{90}{100}$	$\frac{90^2}{100}$
91	$\frac{91}{100}$	$\frac{100}{100}$	10	$\frac{910}{100}$		$\frac{9}{100}$	$\frac{9}{100}$	$\frac{91}{100}$	$\frac{91^2}{100}$
92	$\frac{92}{100}$	$\frac{100}{100}$	10	$\frac{920}{100}$		$\frac{8}{100}$	$\frac{8}{100}$	$\frac{92}{100}$	$\frac{92^2}{100}$
93	$\frac{93}{100}$	$\frac{100}{100}$	10	$\frac{930}{100}$		$\frac{7}{100}$	$\frac{7}{100}$	$\frac{93}{100}$	$\frac{93^2}{100}$
94	$\frac{94}{100}$	$\frac{100}{100}$	10	$\frac{940}{100}$		$\frac{6}{100}$	$\frac{6}{100}$	$\frac{94}{100}$	$\frac{94^2}{100}$
95	$\frac{95}{100}$	$\frac{100}{100}$	10	$\frac{950}{100}$		$\frac{5}{100}$	$\frac{5}{100}$	$\frac{95}{100}$	$\frac{95^2}{100}$
96	$\frac{96}{100}$	$\frac{100}{100}$	10	$\frac{960}{100}$		$\frac{4}{100}$	$\frac{4}{100}$	$\frac{96}{100}$	$\frac{96^2}{100}$
97	$\frac{97}{100}$	$\frac{100}{100}$	10	$\frac{970}{100}$		$\frac{3}{100}$	$\frac{3}{100}$	$\frac{97}{100}$	$\frac{97^2}{100}$
98	$\frac{98}{100}$	$\frac{100}{100}$	10	$\frac{980}{100}$		$\frac{2}{100}$	$\frac{2}{100}$	$\frac{98}{100}$	$\frac{98^2}{100}$
99	$\frac{99}{100}$	$\frac{100}{100}$	10	$\frac{990}{100}$		$\frac{1}{100}$	$\frac{1}{100}$	$\frac{99}{100}$	$\frac{99^2}{100}$
100	$\frac{100}{100}$	$\frac{100}{100}$	10	$\frac{1000}{100}$		$\frac{0}{100}$	$\frac{0}{100}$	$\frac{100}{100}$	$\frac{100^2}{100}$

TABLE II.

SECTION I.] [CLASS I.

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	$\frac{1}{100}$	$\frac{100}{100}$	10	$\frac{10}{100}$		$\frac{99}{100}$	$\frac{99}{100}$	$\frac{1}{100}$	$\frac{1^2}{100}$
2	$\frac{1}{99}$	$\frac{99}{99}$	10	$\frac{10}{99}$		$\frac{98}{99}$	$\frac{98}{99}$	$\frac{1}{99}$	$\frac{1^2}{99}$
3	$\frac{1}{98}$	$\frac{98}{98}$	10	$\frac{10}{98}$		$\frac{97}{98}$	$\frac{97}{98}$	$\frac{1}{98}$	$\frac{1^2}{98}$
4	$\frac{1}{97}$	$\frac{97}{97}$	10	$\frac{10}{97}$		$\frac{96}{97}$	$\frac{96}{97}$	$\frac{1}{97}$	$\frac{1^2}{97}$
5	$\frac{1}{96}$	$\frac{96}{96}$	10	$\frac{10}{96}$		$\frac{95}{96}$	$\frac{95}{96}$	$\frac{1}{96}$	$\frac{1^2}{96}$
6	$\frac{1}{95}$	$\frac{95}{95}$	10	$\frac{10}{95}$		$\frac{94}{95}$	$\frac{94}{95}$	$\frac{1}{95}$	$\frac{1^2}{95}$
7	$\frac{1}{94}$	$\frac{94}{94}$	10	$\frac{10}{94}$		$\frac{93}{94}$	$\frac{93}{94}$	$\frac{1}{94}$	$\frac{1^2}{94}$
8	$\frac{1}{93}$	$\frac{93}{93}$	10	$\frac{10}{93}$		$\frac{92}{93}$	$\frac{92}{93}$	$\frac{1}{93}$	$\frac{1^2}{93}$
9	$\frac{1}{92}$	$\frac{92}{92}$	10	$\frac{10}{92}$		$\frac{91}{92}$	$\frac{91}{92}$	$\frac{1}{92}$	$\frac{1^2}{92}$
10	$\frac{1}{91}$	$\frac{91}{91}$	10	$\frac{10}{91}$		$\frac{90}{91}$	$\frac{90}{91}$	$\frac{1}{91}$	$\frac{1^2}{91}$
11	$\frac{1}{90}$	$\frac{90}{90}$	10	$\frac{10}{90}$		$\frac{89}{90}$	$\frac{89}{90}$	$\frac{1}{90}$	$\frac{1^2}{90}$
12	$\frac{1}{89}$	$\frac{89}{89}$	10	$\frac{10}{89}$		$\frac{88}{89}$	$\frac{88}{89}$	$\frac{1}{89}$	$\frac{1^2}{89}$
13	$\frac{1}{88}$	$\frac{88}{88}$	10	$\frac{10}{88}$		$\frac{87}{88}$	$\frac{87}{88}$	$\frac{1}{88}$	$\frac{1^2}{88}$
14	$\frac{1}{87}$	$\frac{87}{87}$	10	$\frac{10}{87}$		$\frac{86}{87}$	$\frac{86}{87}$	$\frac{1}{87}$	$\frac{1^2}{87}$
15	$\frac{1}{86}$	$\frac{86}{86}$	10	$\frac{10}{86}$		$\frac{85}{86}$	$\frac{85}{86}$	$\frac{1}{86}$	$\frac{1^2}{86}$
16	$\frac{1}{85}$	$\frac{85}{85}$	10	$\frac{10}{85}$		$\frac{84}{85}$	$\frac{84}{85}$	$\frac{1}{85}$	$\frac{1^2}{85}$
17	$\frac{1}{84}$	$\frac{84}{84}$	10	$\frac{10}{84}$		$\frac{83}{84}$	$\frac{83}{84}$	$\frac{1}{84}$	$\frac{1^2}{84}$
18	$\frac{1}{83}$	$\frac{83}{83}$	10	$\frac{10}{83}$		$\frac{82}{83}$	$\frac{82}{83}$	$\frac{1}{83}$	$\frac{1^2}{83}$
19	$\frac{1}{82}$	$\frac{82}{82}$	10	$\frac{10}{82}$		$\frac{81}{82}$	$\frac{81}{82}$	$\frac{1}{82}$	$\frac{1^2}{82}$
20	$\frac{1}{81}$	$\frac{81}{81}$	10	$\frac{10}{81}$		$\frac{80}{81}$	$\frac{80}{81}$	$\frac{1}{81}$	$\frac{1^2}{81}$
21	$\frac{1}{80}$	$\frac{80}{80}$	10	$\frac{10}{80}$		$\frac{79}{80}$	$\frac{79}{80}$	$\frac{1}{80}$	$\frac{1^2}{80}$
22	$\frac{1}{79}$	$\frac{79}{79}$	10	$\frac{10}{79}$		$\frac{78}{79}$	$\frac{78}{79}$	$\frac{1}{79}$	$\frac{1^2}{79}$
23	$\frac{1}{78}$	$\frac{78}{78}$	10	$\frac{10}{78}$		$\frac{77}{78}$	$\frac{77}{78}$	$\frac{1}{78}$	$\frac{1^2}{78}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	$\frac{1}{77}$	$\frac{77}{77}$	10	$\frac{10}{77}$		$\frac{76}{77}$	$\frac{76}{77}$	$\frac{1}{77}$	$\frac{1^2}{77}$
25	$\frac{1}{76}$	$\frac{76}{76}$	10	$\frac{10}{76}$		$\frac{75}{76}$	$\frac{75}{76}$	$\frac{1}{76}$	$\frac{1^2}{76}$
26	$\frac{1}{75}$	$\frac{75}{75}$	10	$\frac{10}{75}$		$\frac{74}{75}$	$\frac{74}{75}$	$\frac{1}{75}$	$\frac{1^2}{75}$
27	$\frac{1}{74}$	$\frac{74}{74}$	10	$\frac{10}{74}$		$\frac{73}{74}$	$\frac{73}{74}$	$\frac{1}{74}$	$\frac{1^2}{74}$
28	$\frac{1}{73}$	$\frac{73}{73}$	10	$\frac{10}{73}$		$\frac{72}{73}$	$\frac{72}{73}$	$\frac{1}{73}$	$\frac{1^2}{73}$
29	$\frac{1}{72}$	$\frac{72}{72}$	10	$\frac{10}{72}$		$\frac{71}{72}$	$\frac{71}{72}$	$\frac{1}{72}$	$\frac{1^2}{72}$
30	$\frac{1}{71}$	$\frac{71}{71}$	10	$\frac{10}{71}$		$\frac{70}{71}$	$\frac{70}{71}$	$\frac{1}{71}$	$\frac{1^2}{71}$
31	$\frac{1}{70}$	$\frac{70}{70}$	10	$\frac{10}{70}$		$\frac{69}{70}$	$\frac{69}{70}$	$\frac{1}{70}$	$\frac{1^2}{70}$
32	$\frac{1}{69}$	$\frac{69}{69}$	10	$\frac{10}{69}$		$\frac{68}{69}$	$\frac{68}{69}$	$\frac{1}{69}$	$\frac{1^2}{69}$
33	$\frac{1}{68}$	$\frac{68}{68}$	10	$\frac{10}{68}$		$\frac{67}{68}$	$\frac{67}{68}$	$\frac{1}{68}$	$\frac{1^2}{68}$
34	$\frac{1}{67}$	$\frac{67}{67}$	10	$\frac{10}{67}$		$\frac{66}{67}$	$\frac{66}{67}$	$\frac{1}{67}$	$\frac{1^2}{67}$
35	$\frac{1}{66}$	$\frac{66}{66}$	10	$\frac{10}{66}$		$\frac{65}{66}$	$\frac{65}{66}$	$\frac{1}{66}$	$\frac{1^2}{66}$
36	$\frac{1}{65}$	$\frac{65}{65}$	10	$\frac{10}{65}$		$\frac{64}{65}$	$\frac{64}{65}$	$\frac{1}{65}$	$\frac{1^2}{65}$
37	$\frac{1}{64}$	$\frac{64}{64}$	10	$\frac{10}{64}$		$\frac{63}{64}$	$\frac{63}{64}$	$\frac{1}{64}$	$\frac{1^2}{64}$
38	$\frac{1}{63}$	$\frac{63}{63}$	10	$\frac{10}{63}$		$\frac{62}{63}$	$\frac{62}{63}$	$\frac{1}{63}$	$\frac{1^2}{63}$
39	$\frac{1}{62}$	$\frac{62}{62}$	10	$\frac{10}{62}$		$\frac{61}{62}$	$\frac{61}{62}$	$\frac{1}{62}$	$\frac{1^2}{62}$
40	$\frac{1}{61}$	$\frac{61}{61}$	10	$\frac{10}{61}$		$\frac{60}{61}$	$\frac{60}{61}$	$\frac{1}{61}$	$\frac{1^2}{61}$
41	$\frac{1}{60}$	$\frac{60}{60}$	10	$\frac{10}{60}$		$\frac{59}{60}$	$\frac{59}{60}$	$\frac{1}{60}$	$\frac{1^2}{60}$
42	$\frac{1}{59}$	$\frac{59}{59}$	10	$\frac{10}{59}$		$\frac{58}{59}$	$\frac{58}{59}$	$\frac{1}{59}$	$\frac{1^2}{59}$
43	$\frac{1}{58}$	$\frac{58}{58}$	10	$\frac{10}{58}$		$\frac{57}{58}$	$\frac{57}{58}$	$\frac{1}{58}$	$\frac{1^2}{58}$
44	$\frac{1}{57}$	$\frac{57}{57}$	10	$\frac{10}{57}$		$\frac{56}{57}$	$\frac{56}{57}$	$\frac{1}{57}$	$\frac{1^2}{57}$
45	$\frac{1}{56}$	$\frac{56}{56}$	10	$\frac{10}{56}$		$\frac{55}{56}$	$\frac{55}{56}$	$\frac{1}{56}$	$\frac{1^2}{56}$
46	$\frac{1}{55}$	$\frac{55}{55}$	10	$\frac{10}{55}$		$\frac{54}{55}$	$\frac{54}{55}$	$\frac{1}{55}$	$\frac{1^2}{55}$
47	$\frac{1}{54}$	$\frac{54}{54}$	10	$\frac{10}{54}$		$\frac{53}{54}$	$\frac{53}{54}$	$\frac{1}{54}$	$\frac{1^2}{54}$
48	$\frac{1}{53}$	$\frac{53}{53}$	10	$\frac{10}{53}$		$\frac{52}{53}$	$\frac{52}{53}$	$\frac{1}{53}$	$\frac{1^2}{53}$
49	$\frac{1}{52}$	$\frac{52}{52}$	10	$\frac{10}{52}$		$\frac{51}{52}$	$\frac{51}{52}$	$\frac{1}{52}$	$\frac{1^2}{52}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	$\frac{1}{51}$	$\frac{51}{51}$	10	$\frac{10}{51}$		$\frac{50}{51}$	$\frac{50}{51}$	$\frac{1}{51}$	$\frac{1^2}{51}$
51	$\frac{1}{50}$	$\frac{50}{50}$	10	$\frac{10}{50}$		$\frac{49}{50}$	$\frac{49}{50}$	$\frac{1}{50}$	$\frac{1^2}{50}$
52	$\frac{1}{49}$	$\frac{49}{49}$	10	$\frac{10}{49}$		$\frac{48}{49}$	$\frac{48}{49}$	$\frac{1}{49}$	$\frac{1^2}{49}$
53	$\frac{1}{48}$	$\frac{48}{48}$	10	$\frac{10}{48}$		$\frac{47}{48}$	$\frac{47}{48}$	$\frac{1}{48}$	$\frac{1^2}{48}$
54	$\frac{1}{47}$	$\frac{47}{47}$	10	$\frac{10}{47}$		$\frac{46}{47}$	$\frac{46}{47}$	$\frac{1}{47}$	$\frac{1^2}{47}$
55	$\frac{1}{46}$	$\frac{46}{46}$	10	$\frac{10}{46}$		$\frac{45}{46}$	$\frac{45}{46}$	$\frac{1}{46}$	$\frac{1^2}{46}$
56	$\frac{1}{45}$	$\frac{45}{45}$	10	$\frac{10}{45}$		$\frac{44}{45}$	$\frac{44}{45}$	$\frac{1}{45}$	$\frac{1^2}{45}$
57	$\frac{1}{44}$	$\frac{44}{44}$	10	$\frac{10}{44}$		$\frac{43}{44}$	$\frac{43}{44}$	$\frac{1}{44}$	$\frac{1^2}{44}$
58	$\frac{1}{43}$	$\frac{43}{43}$	10	$\frac{10}{43}$		$\frac{42}{43}$	$\frac{42}{43}$	$\frac{1}{43}$	$\frac{1^2}{43}$
59	$\frac{1}{42}$	$\frac{42}{42}$	10	$\frac{10}{42}$		$\frac{41}{42}$	$\frac{41}{42}$	$\frac{1}{42}$	$\frac{1^2}{42}$
60	$\frac{1}{41}$	$\frac{41}{41}$	10	$\frac{10}{41}$		$\frac{40}{41}$	$\frac{40}{41}$	$\frac{1}{41}$	$\frac{1^2}{41}$
61	$\frac{1}{40}$	$\frac{40}{40}$	10	$\frac{10}{40}$		$\frac{39}{40}$	$\frac{39}{40}$	$\frac{1}{40}$	$\frac{1^2}{40}$
62	$\frac{1}{39}$	$\frac{39}{39}$	10	$\frac{10}{39}$		$\frac{38}{39}$	$\frac{38}{39}$	$\frac{1}{39}$	$\frac{1^2}{39}$
63	$\frac{1}{38}$	$\frac{38}{38}$	10	$\frac{10}{38}$		$\frac{37}{38}$	$\frac{37}{38}$	$\frac{1}{38}$	$\frac{1^2}{38}$
64	$\frac{1}{37}$	$\frac{37}{37}$	10	$\frac{10}{37}$		$\frac{36}{37}$	$\frac{36}{37}$	$\frac{1}{37}$	$\frac{1^2}{37}$
65	$\frac{1}{36}$	$\frac{36}{36}$	10	$\frac{10}{36}$		$\frac{35}{36}$	$\frac{35}{36}$	$\frac{1}{36}$	$\frac{1^2}{36}$
66	$\frac{1}{35}$	$\frac{35}{35}$	10	$\frac{10}{35}$		$\frac{34}{35}$	$\frac{34}{35}$	$\frac{1}{35}$	$\frac{1^2}{35}$
67	$\frac{1}{34}$	$\frac{34}{34}$	10	$\frac{10}{34}$		$\frac{33}{34}$	$\frac{33}{34}$	$\frac{1}{34}$	$\frac{1^2}{34}$
68	$\frac{1}{33}$	$\frac{33}{33}$	10	$\frac{10}{33}$		$\frac{32}{33}$	$\frac{32}{33}$	$\frac{1}{33}$	$\frac{1^2}{33}$
69	$\frac{1}{32}$	$\frac{32}{32}$	10	$\frac{10}{32}$		$\frac{31}{32}$	$\frac{31}{32}$	$\frac{1}{32}$	$\frac{1^2}{32}$
70	$\frac{1}{31}$	$\frac{31}{31}$	10	$\frac{10}{31}$		$\frac{30}{31}$	$\frac{30}{31}$	$\frac{1}{31}$	$\frac{1^2}{31}$
71	$\frac{1}{30}$	$\frac{30}{30}$	10	$\frac{10}{30}$		$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{30}$	$\frac{1^2}{30}$
72	$\frac{1}{29}$	$\frac{29}{29}$	10	$\frac{10}{29}$		$\frac{28}{29}$	$\frac{28}{29}$	$\frac{1}{29}$	$\frac{1^2}{29}$
73	$\frac{1}{28}$	$\frac{28}{28}$	10	$\frac{10}{28}$		$\frac{27}{28}$	$\frac{27}{28}$	$\frac{1}{28}$	$\frac{1^2}{28}$
74	$\frac{1}{27}$	$\frac{27}{27}$	10	$\frac{10}{27}$		$\frac{26}{27}$	$\frac{26}{27}$	$\frac{1}{27}$	$\frac{1^2}{27}$
75	$\frac{1}{26}$	$\frac{26}{26}$	10	$\frac{10}{26}$		$\frac{25}{26}$	$\frac{25}{26}$	$\frac{1}{26}$	$\frac{1^2}{26}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	$\frac{1}{26}$	$\frac{25}{26}$	10	$\frac{10}{26}$		$\frac{24}{26}$	$\frac{24}{26}$	$\frac{1}{26}$	$\frac{1^2}{26}$
77	$\frac{1}{24}$	$\frac{24}{24}$	10	$\frac{10}{24}$		$\frac{23}{24}$	$\frac{23}{24}$	$\frac{1}{24}$	$\frac{1^2}{24}$
78	$\frac{1}{23}$	$\frac{23}{23}$	10	$\frac{10}{23}$		$\frac{22}{23}$	$\frac{22}{23}$	$\frac{1}{23}$	$\frac{1^2}{23}$
79	$\frac{1}{22}$	$\frac{22}{22}$	10	$\frac{10}{22}$		$\frac{21}{22}$	$\frac{21}{22}$	$\frac{1}{22}$	$\frac{1^2}{22}$
80	$\frac{1}{21}$	$\frac{21}{21}$	10	$\frac{10}{21}$		$\frac{20}{21}$	$\frac{20}{21}$	$\frac{1}{21}$	$\frac{1^2}{21}$
81	$\frac{1}{20}$	$\frac{20}{20}$	10	$\frac{10}{20}$		$\frac{19}{20}$	$\frac{19}{20}$	$\frac{1}{20}$	$\frac{1^2}{20}$
82	$\frac{1}{19}$	$\frac{19}{19}$	10	$\frac{10}{19}$		$\frac{18}{19}$	$\frac{18}{19}$	$\frac{1}{19}$	$\frac{1^2}{19}$
83	$\frac{1}{18}$	$\frac{18}{18}$	10	$\frac{10}{18}$		$\frac{17}{18}$	$\frac{17}{18}$	$\frac{1}{18}$	$\frac{1^2}{18}$
84	$\frac{1}{17}$	$\frac{17}{17}$	10	$\frac{10}{17}$		$\frac{16}{17}$	$\frac{16}{17}$	$\frac{1}{17}$	$\frac{1^2}{17}$
85	$\frac{1}{16}$	$\frac{16}{16}$	10	$\frac{10}{16}$		$\frac{15}{16}$	$\frac{15}{16}$	$\frac{1}{16}$	$\frac{1^2}{16}$
86	$\frac{1}{15}$	$\frac{15}{15}$	10	$\frac{10}{15}$		$\frac{14}{15}$	$\frac{14}{15}$	$\frac{1}{15}$	$\frac{1^2}{15}$
87	$\frac{1}{14}$	$\frac{14}{14}$	10	$\frac{10}{14}$		$\frac{13}{14}$	$\frac{13}{14}$	$\frac{1}{14}$	$\frac{1^2}{14}$
88	$\frac{1}{13}$	$\frac{13}{13}$	10	$\frac{10}{13}$		$\frac{12}{13}$	$\frac{12}{13}$	$\frac{1}{13}$	$\frac{1^2}{13}$
89	$\frac{1}{12}$	$\frac{12}{12}$	10	$\frac{10}{12}$		$\frac{11}{12}$	$\frac{11}{12}$	$\frac{1}{12}$	$\frac{1^2}{12}$
90	$\frac{1}{11}$	$\frac{11}{11}$	10	$\frac{10}{11}$		$\frac{10}{11}$	$\frac{10}{11}$	$\frac{1}{11}$	$\frac{1^2}{11}$
91	$\frac{1}{10}$	$\frac{10}{10}$	10	$\frac{10}{10}$		$\frac{9}{10}$	$\frac{9}{10}$	$\frac{1}{10}$	$\frac{1^2}{10}$
92	$\frac{1}{9}$	$\frac{9}{9}$	10	$\frac{10}{9}$		$\frac{8}{9}$	$\frac{8}{9}$	$\frac{1}{9}$	$\frac{1^2}{9}$
93	$\frac{1}{8}$	$\frac{8}{8}$	10	$\frac{10}{8}$		$\frac{7}{8}$	$\frac{7}{8}$	$\frac{1}{8}$	$\frac{1^2}{8}$
94	$\frac{1}{7}$	$\frac{7}{7}$	10	$\frac{10}{7}$		$\frac{6}{7}$	$\frac{6}{7}$	$\frac{1}{7}$	$\frac{1^2}{7}$
95	$\frac{1}{6}$	$\frac{6}{6}$	10	$\frac{10}{6}$		$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1^2}{6}$
96	$\frac{1}{5}$	$\frac{5}{5}$	10	$\frac{10}{5}$		$\frac{4}{5}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{1^2}{5}$
97	$\frac{1}{4}$	$\frac{4}{4}$	10	$\frac{10}{4}$		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1^2}{4}$
98	$\frac{1}{3}$	$\frac{3}{3}$	10	$\frac{10}{3}$		$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1^2}{3}$
99	$\frac{1}{2}$	$\frac{2}{2}$	10	$\frac{10}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1^2}{2}$
100	$\frac{1}{1}$	$\frac{1}{1}$	10	$\frac{10}{1}$		$\frac{0}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{1^2}{1}$

TABLE III.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	1	1	10	10		$\frac{0}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{1^2}{1}$
2	1	2	10	10		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1^2}{2}$
3	1	3	10	10		$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1^2}{3}$
4	1	4	10	10		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1^2}{4}$
5	1	5	10	10		$\frac{4}{5}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{1^2}{5}$
6	1	6	10	10		$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1^2}{6}$
7	1	7	10	10		$\frac{6}{7}$	$\frac{6}{7}$	$\frac{1}{7}$	$\frac{1^2}{7}$
8	1	8	10	10		$\frac{7}{8}$	$\frac{7}{8}$	$\frac{1}{8}$	$\frac{1^2}{8}$
9	1	9	10	10		$\frac{8}{9}$	$\frac{8}{9}$	$\frac{1}{9}$	$\frac{1^2}{9}$
10	1	10	10	10		$\frac{9}{10}$	$\frac{9}{10}$	$\frac{1}{10}$	$\frac{1^2}{10}$
11	1	11	10	10		$\frac{10}{11}$	$\frac{10}{11}$	$\frac{1}{11}$	$\frac{1^2}{11}$
12	1	12	10	10		$\frac{11}{12}$	$\frac{11}{12}$	$\frac{1}{12}$	$\frac{1^2}{12}$
13	1	13	10	10		$\frac{12}{13}$	$\frac{12}{13}$	$\frac{1}{13}$	$\frac{1^2}{13}$
14	1	14	10	10		$\frac{13}{14}$	$\frac{13}{14}$	$\frac{1}{14}$	$\frac{1^2}{14}$
15	1	15	10	10		$\frac{14}{15}$	$\frac{14}{15}$	$\frac{1}{15}$	$\frac{1^2}{15}$
16	1	16	10	10		$\frac{15}{16}$	$\frac{15}{16}$	$\frac{1}{16}$	$\frac{1^2}{16}$
17	1	17	10	10		$\frac{16}{17}$	$\frac{16}{17}$	$\frac{1}{17}$	$\frac{1^2}{17}$
18	1	18	10	10		$\frac{17}{18}$	$\frac{17}{18}$	$\frac{1}{18}$	$\frac{1^2}{18}$
19	1	19	10	10		$\frac{18}{19}$	$\frac{18}{19}$	$\frac{1}{19}$	$\frac{1^2}{19}$
20	1	20	10	10		$\frac{19}{20}$	$\frac{19}{20}$	$\frac{1}{20}$	$\frac{1^2}{20}$
21	1	21	10	10		$\frac{20}{21}$	$\frac{20}{21}$	$\frac{1}{21}$	$\frac{1^2}{21}$
22	1	22	10	10		$\frac{21}{22}$	$\frac{21}{22}$	$\frac{1}{22}$	$\frac{1^2}{22}$
23	1	23	10	10		$\frac{22}{23}$	$\frac{22}{23}$	$\frac{1}{23}$	$\frac{1^2}{23}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	1	24	10	10		$\frac{23}{24}$	$\frac{23}{24}$	$\frac{1}{24}$	$\frac{1}{24}$
25	1	25	10	10		$\frac{24}{25}$	$\frac{24}{25}$	$\frac{1}{25}$	$\frac{1}{25^2}$
26	1	26	10	10		$\frac{25}{26}$	$\frac{25}{26}$	$\frac{1}{26}$	$\frac{1}{26^2}$
27	1	27	10	10		$\frac{26}{27}$	$\frac{26}{27}$	$\frac{1}{27}$	$\frac{1}{27^2}$
28	1	28	10	10		$\frac{27}{28}$	$\frac{27}{28}$	$\frac{1}{28}$	$\frac{1}{28^2}$
29	1	29	10	10		$\frac{28}{29}$	$\frac{28}{29}$	$\frac{1}{29}$	$\frac{1}{29^2}$
30	1	30	10	10		$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{30}$	$\frac{1}{30^2}$
31	1	31	10	10		$\frac{30}{31}$	$\frac{30}{31}$	$\frac{1}{31}$	$\frac{1}{31^2}$
32	1	32	10	10		$\frac{31}{32}$	$\frac{31}{32}$	$\frac{1}{32}$	$\frac{1}{32^2}$
33	1	33	10	10		$\frac{32}{33}$	$\frac{32}{33}$	$\frac{1}{33}$	$\frac{1}{33^2}$
34	1	34	10	10		$\frac{33}{34}$	$\frac{33}{34}$	$\frac{1}{34}$	$\frac{1}{34^2}$
35	1	35	10	10		$\frac{34}{35}$	$\frac{34}{35}$	$\frac{1}{35}$	$\frac{1}{35^2}$
36	1	36	10	10		$\frac{35}{36}$	$\frac{35}{36}$	$\frac{1}{36}$	$\frac{1}{36^2}$
37	1	37	10	10		$\frac{36}{37}$	$\frac{36}{37}$	$\frac{1}{37}$	$\frac{1}{37^2}$
38	1	38	10	10		$\frac{37}{38}$	$\frac{37}{38}$	$\frac{1}{38}$	$\frac{1}{38^2}$
39	1	39	10	10		$\frac{38}{39}$	$\frac{38}{39}$	$\frac{1}{39}$	$\frac{1}{39^2}$
40	1	40	10	10		$\frac{39}{40}$	$\frac{39}{40}$	$\frac{1}{40}$	$\frac{1}{40^2}$
41	1	41	10	10		$\frac{40}{41}$	$\frac{40}{41}$	$\frac{1}{41}$	$\frac{1}{41^2}$
42	1	42	10	10		$\frac{41}{42}$	$\frac{41}{42}$	$\frac{1}{42}$	$\frac{1}{42^2}$
43	1	43	10	10		$\frac{42}{43}$	$\frac{42}{43}$	$\frac{1}{43}$	$\frac{1}{43^2}$
44	1	44	10	10		$\frac{43}{44}$	$\frac{43}{44}$	$\frac{1}{44}$	$\frac{1}{44^2}$
45	1	45	10	10		$\frac{44}{45}$	$\frac{44}{45}$	$\frac{1}{45}$	$\frac{1}{45^2}$
46	1	46	10	10		$\frac{45}{46}$	$\frac{45}{46}$	$\frac{1}{46}$	$\frac{1}{46^2}$
47	1	47	10	10		$\frac{46}{47}$	$\frac{46}{47}$	$\frac{1}{47}$	$\frac{1}{47^2}$
48	1	48	10	10		$\frac{47}{48}$	$\frac{47}{48}$	$\frac{1}{48}$	$\frac{1}{48^2}$
49	1	49	10	10		$\frac{48}{49}$	$\frac{48}{49}$	$\frac{1}{49}$	$\frac{1}{49^2}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	1	50	10	10		$\frac{49}{50}$	$\frac{49}{50}$	$\frac{1}{50}$	$\frac{1^2}{50}$
51	1	51	10	10		$\frac{50}{51}$	$\frac{50}{51}$	$\frac{1}{51}$	$\frac{1^2}{51}$
52	1	52	10	10		$\frac{51}{52}$	$\frac{51}{52}$	$\frac{1}{52}$	$\frac{1^2}{52}$
53	1	53	10	10		$\frac{52}{53}$	$\frac{52}{53}$	$\frac{1}{53}$	$\frac{1^2}{53}$
54	1	54	10	10		$\frac{53}{54}$	$\frac{53}{54}$	$\frac{1}{54}$	$\frac{1^2}{54}$
55	1	55	10	10		$\frac{54}{55}$	$\frac{54}{55}$	$\frac{1}{55}$	$\frac{1^2}{55}$
56	1	56	10	10		$\frac{55}{56}$	$\frac{55}{56}$	$\frac{1}{56}$	$\frac{1^2}{56}$
57	1	57	10	10		$\frac{56}{57}$	$\frac{56}{57}$	$\frac{1}{57}$	$\frac{1^2}{57}$
58	1	58	10	10		$\frac{57}{58}$	$\frac{57}{58}$	$\frac{1}{58}$	$\frac{1^2}{58}$
59	1	59	10	10		$\frac{58}{59}$	$\frac{58}{59}$	$\frac{1}{59}$	$\frac{1^2}{59}$
60	1	60	10	10		$\frac{59}{60}$	$\frac{59}{60}$	$\frac{1}{60}$	$\frac{1^2}{60}$
61	1	61	10	10		$\frac{60}{61}$	$\frac{60}{61}$	$\frac{1}{61}$	$\frac{1^2}{61}$
62	1	62	10	10		$\frac{61}{62}$	$\frac{61}{62}$	$\frac{1}{62}$	$\frac{1^2}{62}$
63	1	63	10	10		$\frac{62}{63}$	$\frac{62}{63}$	$\frac{1}{63}$	$\frac{1^2}{63}$
64	1	64	10	10		$\frac{63}{64}$	$\frac{63}{64}$	$\frac{1}{64}$	$\frac{1^2}{64}$
65	1	65	10	10		$\frac{64}{65}$	$\frac{64}{65}$	$\frac{1}{65}$	$\frac{1^2}{65}$
66	1	66	10	10		$\frac{65}{66}$	$\frac{65}{66}$	$\frac{1}{66}$	$\frac{1^2}{66}$
67	1	67	10	10		$\frac{66}{67}$	$\frac{66}{67}$	$\frac{1}{67}$	$\frac{1^2}{67}$
68	1	68	10	10		$\frac{67}{68}$	$\frac{67}{68}$	$\frac{1}{68}$	$\frac{1^2}{68}$
69	1	69	10	10		$\frac{68}{69}$	$\frac{68}{69}$	$\frac{1}{69}$	$\frac{1^2}{69}$
70	1	70	10	10		$\frac{69}{70}$	$\frac{69}{70}$	$\frac{1}{70}$	$\frac{1^2}{70}$
71	1	71	10	10		$\frac{70}{71}$	$\frac{70}{71}$	$\frac{1}{71}$	$\frac{1^2}{71}$
72	1	72	10	10		$\frac{71}{72}$	$\frac{71}{72}$	$\frac{1}{72}$	$\frac{1^2}{72}$
73	1	73	10	10		$\frac{72}{73}$	$\frac{72}{73}$	$\frac{1}{73}$	$\frac{1^2}{73}$
74	1	74	10	10		$\frac{73}{74}$	$\frac{73}{74}$	$\frac{1}{74}$	$\frac{1^2}{74}$
75	1	75	10	10		$\frac{74}{75}$	$\frac{74}{75}$	$\frac{1}{75}$	$\frac{1^2}{75}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	1	76	10	10		$\frac{75}{76}$	$\frac{75}{76}$	$\frac{1}{76}$	$\frac{1^2}{76}$
77	1	77	10	10		$\frac{76}{77}$	$\frac{76}{77}$	$\frac{1}{77}$	$\frac{1^2}{77}$
78	1	78	10	10		$\frac{77}{78}$	$\frac{77}{78}$	$\frac{1}{78}$	$\frac{1^2}{78}$
79	1	79	10	10		$\frac{78}{79}$	$\frac{78}{79}$	$\frac{1}{79}$	$\frac{1^2}{79}$
80	1	80	10	10		$\frac{79}{80}$	$\frac{79}{80}$	$\frac{1}{80}$	$\frac{1^2}{80}$
81	1	81	10	10		$\frac{80}{81}$	$\frac{80}{81}$	$\frac{1}{81}$	$\frac{1^2}{81}$
82	1	82	10	10		$\frac{81}{82}$	$\frac{81}{82}$	$\frac{1}{82}$	$\frac{1^2}{82}$
83	1	83	10	10		$\frac{82}{83}$	$\frac{82}{83}$	$\frac{1}{83}$	$\frac{1^2}{83}$
84	1	84	10	10		$\frac{83}{84}$	$\frac{83}{84}$	$\frac{1}{84}$	$\frac{1^2}{84}$
85	1	85	10	10		$\frac{84}{85}$	$\frac{84}{85}$	$\frac{1}{85}$	$\frac{1^2}{85}$
86	1	86	10	10		$\frac{85}{86}$	$\frac{85}{86}$	$\frac{1}{86}$	$\frac{1^2}{86}$
87	1	87	10	10		$\frac{86}{87}$	$\frac{86}{87}$	$\frac{1}{87}$	$\frac{1^2}{87}$
88	1	88	10	10		$\frac{87}{88}$	$\frac{87}{88}$	$\frac{1}{88}$	$\frac{1^2}{88}$
89	1	89	10	10		$\frac{88}{89}$	$\frac{88}{89}$	$\frac{1}{89}$	$\frac{1^2}{89}$
90	1	90	10	10		$\frac{89}{90}$	$\frac{89}{90}$	$\frac{1}{90}$	$\frac{1^2}{90}$
91	1	91	10	10		$\frac{90}{91}$	$\frac{90}{91}$	$\frac{1}{91}$	$\frac{1^2}{91}$
92	1	92	10	10		$\frac{91}{92}$	$\frac{91}{92}$	$\frac{1}{92}$	$\frac{1^2}{92}$
93	1	93	10	10		$\frac{92}{93}$	$\frac{92}{93}$	$\frac{1}{93}$	$\frac{1^2}{93}$
94	1	94	10	10		$\frac{93}{94}$	$\frac{93}{94}$	$\frac{1}{94}$	$\frac{1^2}{94}$
95	1	95	10	10		$\frac{94}{95}$	$\frac{94}{95}$	$\frac{1}{95}$	$\frac{1^2}{95}$
96	1	96	10	10		$\frac{95}{96}$	$\frac{95}{96}$	$\frac{1}{96}$	$\frac{1^2}{96}$
97	1	97	10	10		$\frac{96}{97}$	$\frac{96}{97}$	$\frac{1}{97}$	$\frac{1^2}{97}$
98	1	98	10	10		$\frac{97}{98}$	$\frac{97}{98}$	$\frac{1}{98}$	$\frac{1^2}{98}$
99	1	99	10	10		$\frac{98}{99}$	$\frac{98}{99}$	$\frac{1}{99}$	$\frac{1^2}{99}$
100	1	100	10	10		$\frac{99}{100}$	$\frac{99}{100}$	$\frac{1}{100}$	$\frac{1^2}{100}$

TABLE IV

SECTION I.] [CLASS I.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	2	3	10	20		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2^2}{3}$
2	2	4	10	20		$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2^2}{4}$
3	2	5	10	20		$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{2^2}{5}$
4	2	6	10	20		$\frac{4}{6}$	$\frac{4}{6}$	$\frac{2}{6}$	$\frac{2^2}{6}$
5	2	7	10	20		$\frac{5}{7}$	$\frac{5}{7}$	$\frac{2}{7}$	$\frac{2^2}{7}$
6	2	8	10	20		$\frac{6}{8}$	$\frac{6}{8}$	$\frac{2}{8}$	$\frac{2^2}{8}$
7	2	9	10	20		$\frac{7}{9}$	$\frac{7}{9}$	$\frac{2}{9}$	$\frac{2^2}{9}$
8	2	10	10	20		$\frac{8}{10}$	$\frac{8}{10}$	$\frac{2}{10}$	$\frac{2^2}{10}$
9	2	11	10	20		$\frac{9}{11}$	$\frac{9}{11}$	$\frac{2}{11}$	$\frac{2^2}{11}$
10	2	12	10	20		$\frac{10}{12}$	$\frac{10}{12}$	$\frac{2}{12}$	$\frac{2^2}{12}$
11	2	13	10	20		$\frac{11}{13}$	$\frac{11}{13}$	$\frac{2}{13}$	$\frac{2^2}{13}$
12	2	14	10	20		$\frac{12}{14}$	$\frac{12}{14}$	$\frac{2}{14}$	$\frac{2^2}{14}$
13	2	15	10	20		$\frac{13}{15}$	$\frac{13}{15}$	$\frac{2}{15}$	$\frac{2^2}{15}$
14	2	16	10	20		$\frac{14}{16}$	$\frac{14}{16}$	$\frac{2}{16}$	$\frac{2^2}{16}$
15	3	4	10	30		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3^2}{4}$
16	3	5	10	30		$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{3^2}{5}$
17	3	6	10	30		$\frac{3}{6}$	$\frac{3}{6}$	$\frac{3}{6}$	$\frac{3^2}{6}$
18	3	7	10	30		$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{3^2}{7}$
19	3	8	10	30		$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3^2}{8}$
20	3	9	10	30		$\frac{6}{9}$	$\frac{6}{9}$	$\frac{3}{9}$	$\frac{3^2}{9}$
21	3	10	10	30		$\frac{7}{10}$	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{3^2}{10}$
22	3	11	10	30		$\frac{8}{11}$	$\frac{8}{11}$	$\frac{3}{11}$	$\frac{3^2}{11}$
23	3	12	10	30		$\frac{9}{12}$	$\frac{9}{12}$	$\frac{3}{12}$	$\frac{3^2}{12}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	3	13	10	30		$\frac{10}{13}$	$\frac{10}{13}$	$\frac{3}{13}$	$\frac{3^2}{13}$
25	3	14	10	30		$\frac{11}{14}$	$\frac{11}{14}$	$\frac{3}{14}$	$\frac{3^2}{14}$
26	3	15	10	30		$\frac{12}{15}$	$\frac{12}{15}$	$\frac{3}{15}$	$\frac{3^2}{15}$
27	3	16	10	30		$\frac{13}{16}$	$\frac{13}{16}$	$\frac{3}{16}$	$\frac{3^2}{16}$
28	4	5	10	40		$\frac{1}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{4^2}{5}$
29	4	6	10	40		$\frac{2}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4^2}{6}$
30	4	7	10	40		$\frac{3}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4^2}{7}$
31	4	8	10	40		$\frac{4}{8}$	$\frac{4}{8}$	$\frac{4}{8}$	$\frac{4^2}{8}$
32	4	9	10	40		$\frac{5}{9}$	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{4^2}{9}$
33	4	10	10	40		$\frac{6}{10}$	$\frac{6}{10}$	$\frac{4}{10}$	$\frac{4^2}{10}$
34	4	11	10	40		$\frac{7}{11}$	$\frac{7}{11}$	$\frac{4}{11}$	$\frac{4^2}{11}$
35	4	12	10	40		$\frac{8}{12}$	$\frac{8}{12}$	$\frac{4}{12}$	$\frac{4^2}{12}$
36	4	13	10	40		$\frac{9}{13}$	$\frac{9}{13}$	$\frac{4}{13}$	$\frac{4^2}{13}$
37	4	14	10	40		$\frac{10}{14}$	$\frac{10}{14}$	$\frac{4}{14}$	$\frac{4^2}{14}$
38	4	15	10	40		$\frac{11}{15}$	$\frac{11}{15}$	$\frac{4}{15}$	$\frac{4^2}{15}$
39	4	16	10	40		$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	$\frac{4^2}{16}$
40	5	6	10	50		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{5^2}{6}$
41	5	7	10	50		$\frac{2}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{5^2}{7}$
42	5	8	10	50		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5^2}{8}$
43	5	9	10	50		$\frac{4}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{5^2}{9}$
44	5	10	10	50		$\frac{5}{10}$	$\frac{5}{10}$	$\frac{5}{10}$	$\frac{5^2}{10}$
45	5	11	10	50		$\frac{6}{11}$	$\frac{6}{11}$	$\frac{5}{11}$	$\frac{5^2}{11}$
46	5	12	10	50		$\frac{7}{12}$	$\frac{7}{12}$	$\frac{5}{12}$	$\frac{5^2}{12}$
47	5	13	10	50		$\frac{8}{13}$	$\frac{8}{13}$	$\frac{5}{13}$	$\frac{5^2}{13}$
48	5	14	10	50		$\frac{9}{14}$	$\frac{9}{14}$	$\frac{5}{14}$	$\frac{5^2}{14}$
49	5	15	10	50		$\frac{10}{15}$	$\frac{10}{15}$	$\frac{5}{15}$	$\frac{5^2}{15}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	5	16	10	50		$\frac{11}{16}$	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{5^2}{16}$
51	6	7	10	60		$\frac{1}{7}$	$\frac{1}{7}$	$\frac{6}{7}$	$\frac{6^2}{7}$
52	6	8	10	60		$\frac{2}{8}$	$\frac{2}{8}$	$\frac{6}{8}$	$\frac{6^2}{8}$
53	6	9	10	60		$\frac{3}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{6^2}{9}$
54	6	10	10	60		$\frac{4}{10}$	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{6^2}{10}$
55	6	11	10	60		$\frac{5}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{6^2}{11}$
56	6	12	10	60		$\frac{6}{12}$	$\frac{6}{12}$	$\frac{6}{12}$	$\frac{6^2}{12}$
57	6	13	10	60		$\frac{7}{13}$	$\frac{7}{13}$	$\frac{6}{13}$	$\frac{6^2}{13}$
58	6	14	10	60		$\frac{8}{14}$	$\frac{8}{14}$	$\frac{6}{14}$	$\frac{6^2}{14}$
59	6	15	10	60		$\frac{9}{15}$	$\frac{9}{15}$	$\frac{6}{15}$	$\frac{6^2}{15}$
60	6	16	10	60		$\frac{10}{16}$	$\frac{10}{16}$	$\frac{6}{16}$	$\frac{6^2}{16}$
61	7	8	10	70		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{7^2}{8}$
62	7	9	10	70		$\frac{2}{9}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{7^2}{9}$
63	7	10	10	70		$\frac{3}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{7^2}{10}$
64	7	11	10	70		$\frac{4}{11}$	$\frac{4}{11}$	$\frac{7}{11}$	$\frac{7^2}{11}$
65	7	12	10	70		$\frac{5}{12}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{7^2}{12}$
66	7	13	10	70		$\frac{6}{13}$	$\frac{6}{13}$	$\frac{7}{13}$	$\frac{7^2}{13}$
67	7	14	10	70		$\frac{7}{14}$	$\frac{7}{14}$	$\frac{7}{14}$	$\frac{7^2}{14}$
68	7	15	10	70		$\frac{8}{15}$	$\frac{8}{15}$	$\frac{7}{15}$	$\frac{7^2}{15}$
69	7	16	10	70		$\frac{9}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	$\frac{7^2}{16}$
70	8	9	10	80		$\frac{1}{9}$	$\frac{1}{9}$	$\frac{8}{9}$	$\frac{8^2}{9}$
71	8	10	10	80		$\frac{2}{10}$	$\frac{2}{10}$	$\frac{8}{10}$	$\frac{8^2}{10}$
72	8	11	10	80		$\frac{3}{11}$	$\frac{3}{11}$	$\frac{8}{11}$	$\frac{8^2}{11}$
73	8	12	10	80		$\frac{4}{12}$	$\frac{4}{12}$	$\frac{8}{12}$	$\frac{8^2}{12}$
74	8	13	10	80		$\frac{5}{13}$	$\frac{5}{13}$	$\frac{8}{13}$	$\frac{8^2}{13}$
75	8	14	10	80		$\frac{6}{14}$	$\frac{6}{14}$	$\frac{8}{14}$	$\frac{8^2}{14}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	8	15	10	80		$\frac{7}{15}$	$\frac{7}{15}$	$\frac{8}{15}$	$\frac{8^2}{15}$
77	8	16	10	80		$\frac{8}{16}$	$\frac{8}{16}$	$\frac{8}{16}$	$\frac{8^2}{16}$
78	9	10	10	90		$\frac{1}{10}$	$\frac{1}{10}$	$\frac{9}{10}$	$\frac{9^2}{10}$
79	9	11	10	90		$\frac{2}{11}$	$\frac{2}{11}$	$\frac{9}{11}$	$\frac{9^2}{11}$
80	9	12	10	90		$\frac{3}{12}$	$\frac{3}{12}$	$\frac{9}{12}$	$\frac{9^2}{12}$
81	9	13	10	90		$\frac{4}{13}$	$\frac{4}{13}$	$\frac{9}{13}$	$\frac{9^2}{13}$
82	9	14	10	90		$\frac{5}{14}$	$\frac{5}{14}$	$\frac{9}{14}$	$\frac{9^2}{14}$
83	9	15	10	90		$\frac{6}{15}$	$\frac{6}{15}$	$\frac{9}{15}$	$\frac{9^2}{15}$
84	9	16	10	90		$\frac{7}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{9^2}{16}$
85	10	11	10	100		$\frac{1}{11}$	$\frac{1}{11}$	$\frac{10}{11}$	$\frac{10^2}{11}$
86	10	12	10	100		$\frac{2}{12}$	$\frac{2}{12}$	$\frac{10}{12}$	$\frac{10^2}{12}$
87	10	13	10	100		$\frac{3}{13}$	$\frac{3}{13}$	$\frac{10}{13}$	$\frac{10^2}{13}$
88	10	14	10	100		$\frac{4}{14}$	$\frac{4}{14}$	$\frac{10}{14}$	$\frac{10^2}{14}$
89	10	15	10	100		$\frac{5}{15}$	$\frac{5}{15}$	$\frac{10}{15}$	$\frac{10^2}{15}$
90	10	16	10	100		$\frac{6}{16}$	$\frac{6}{16}$	$\frac{10}{16}$	$\frac{10^2}{16}$
91	11	12	10	110		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{11}{12}$	$\frac{11^2}{12}$
92	11	13	10	110		$\frac{2}{13}$	$\frac{2}{13}$	$\frac{11}{13}$	$\frac{11^2}{13}$
93	11	14	10	110		$\frac{3}{14}$	$\frac{3}{14}$	$\frac{11}{14}$	$\frac{11^2}{14}$
94	11	15	10	110		$\frac{4}{15}$	$\frac{4}{15}$	$\frac{11}{15}$	$\frac{11^2}{15}$
95	11	16	10	110		$\frac{5}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{11^2}{16}$
96	12	13	10	120		$\frac{1}{13}$	$\frac{1}{13}$	$\frac{12}{13}$	$\frac{12^2}{13}$
97	12	14	10	120		$\frac{2}{14}$	$\frac{2}{14}$	$\frac{12}{14}$	$\frac{12^2}{14}$
98	12	15	10	120		$\frac{3}{15}$	$\frac{3}{15}$	$\frac{12}{15}$	$\frac{12^2}{15}$
99	12	16	10	120		$\frac{4}{16}$	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12^2}{16}$
100	12	17	10	120		$\frac{5}{17}$	$\frac{5}{17}$	$\frac{12}{17}$	$\frac{12^2}{17}$

SECTION II.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE SECOND CLASS OF CASES, WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS GREATER THAN, OR EQUAL TO, THE WEIGHT OF B; AND IN WHICH B IS AT REST AT THE TIME OF COLLISION.

In every case belonging to the Second Class the results of collision will be determined by the following laws:—

Law 1.—A will move in the same line after as before collision. Both the velocity and momentum of A,—as compared with the velocity and momentum of A before collision,—will be proportional to the difference between the respective weights of B and A.

Law 2.—The momentum of B,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of B to the weight of A.

Law 3.—B will move in the given direction of A. The velocity of B will be equal to the velocity of A before collision.

These laws will give the results of collision in terms of the weight of A; and the results, in every case, may all be expressed by three fractional numbers, namely, that which expresses the difference between the respective weights of B and A; that which expresses the ratio of the weight of B to the weight of A; and that which expresses the sum of the said ratio and difference.

By comparing the laws of Section I. with those of Section II., it will be seen that in the first class of cases, A will always be found, after collision, upon one and the

same side of the point of collision, and B upon the other side; while, in the second class of cases, both A and B will always be found upon one and the same side of the point of collision; that both the velocity and momentum of A, after collision, are determined by the same law in both classes of cases; but that A, in the first class, is reflected back from the point of collision, while, in the second class, A follows on after B; that the momentum of B, in the first class, is proportional to the ratio of the weight of A to the weight of B; while, in the second class, it is proportional to the ratio of the weight of B to the weight of A; that, in the first class, the velocity of B, after collision, is proportional to the square of the ratio of the weight of A to the weight of B; while, in the second class, the velocity of B is equal to the velocity of A, before collision; and, that in the first class of cases, the results of collision are expressed in terms of the weight of B; while, in the second class, the results of collision are expressed in terms of the weight of A.

The following tables have been constructed for the purpose of showing the results of collision in four hundred different cases belonging to the second class. In case No. 20, of the first table, for example, the weight of A is one pound, and the velocity of A, before collision, 10 feet per second. Hence the momentum of A, before collision, is put down in the table as 10, which is the amount that is to be accounted for after collision. The weight of B is $\frac{20}{100}$ of 1 pound. Now, according to Law 1, both the velocity and momentum of A,—as compared with the velocity and momentum of A before collision,—will be proportional to the difference between the respective weights of A and B. As the weight of A, in case No. 20, is 1 pound, or $\frac{100}{100}$, and the weight of B $\frac{20}{100}$; the difference is $\frac{80}{100}$. Hence the velocity and momentum

of A are each put down in the table as $\frac{80}{100}$; the meaning of which is, that the velocity of A will be equal to eighty one-hundredths of the velocity which A had before collision; that is, equal to eighty one-hundredths of ten feet per second; and that the momentum of A will be equal to eighty one-hundredths of the momentum which A had before collision,—that is, equal to eighty one-hundredths of 10.

According to Law 2, the momentum of B, after collision, as compared with the momentum of A before collision, will be proportional to the ratio of the weight of B to the weight of A. In case No. 20, that ratio is $\frac{20}{100}$. Hence the momentum of B, after collision, is put down in the table as $\frac{20}{100}$; and the meaning of it is, that it will be equal to twenty one-hundredths of the momentum which A had before collision: that is, equal to twenty one-hundredths of 10. Now, as the momentum of A, after collision, was equal to $\frac{80}{100}$ of 10, and the momentum of B equal to $\frac{20}{100}$ of 10, all the momentum that was to be accounted for after collision, has been found.

According to Law 3, the velocity of B, after collision, will be equal to the velocity of A before collision. Hence, the velocity of B, after collision, is put down in the table as $\frac{100}{100}$; and the meaning of it is, that it will be equal to one hundred one-hundredths of the velocity which A had before collision.

Now, whatever may be the weight of A, or the weight of B; or, whatever may be the velocity of A before collision; the laws above enunciated will hold good in every case in which the weight of A is equal to, or greater than, the weight of B; and in which B is at rest, with respect to A, at the time of collision.

TABLE I.

No. of the Case.	Weight of A in pounds.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collis
1	1	$\frac{1}{100}$	10	10		$\frac{99}{100}$	$\frac{99}{100}$	$\frac{1}{100}$	$\frac{100}{100}$
2	1	$\frac{2}{100}$	10	10		$\frac{98}{100}$	$\frac{99}{100}$	$\frac{2}{100}$	$\frac{100}{100}$
3	1	$\frac{3}{100}$	10	10		$\frac{97}{100}$	$\frac{97}{100}$	$\frac{3}{100}$	$\frac{100}{100}$
4	1	$\frac{4}{100}$	10	10		$\frac{96}{100}$	$\frac{96}{100}$	$\frac{4}{100}$	$\frac{100}{100}$
5	1	$\frac{5}{100}$	10	10		$\frac{95}{100}$	$\frac{96}{100}$	$\frac{5}{100}$	$\frac{100}{100}$
6	1	$\frac{6}{100}$	10	10		$\frac{94}{100}$	$\frac{94}{100}$	$\frac{6}{100}$	$\frac{100}{100}$
7	1	$\frac{7}{100}$	10	10		$\frac{93}{100}$	$\frac{93}{100}$	$\frac{7}{100}$	$\frac{100}{100}$
8	1	$\frac{8}{100}$	10	10		$\frac{92}{100}$	$\frac{92}{100}$	$\frac{8}{100}$	$\frac{100}{100}$
9	1	$\frac{9}{100}$	10	10		$\frac{91}{100}$	$\frac{91}{100}$	$\frac{9}{100}$	$\frac{100}{100}$
10	1	$\frac{10}{100}$	10	10		$\frac{90}{100}$	$\frac{90}{100}$	$\frac{10}{100}$	$\frac{100}{100}$
11	1	$\frac{11}{100}$	10	10		$\frac{89}{100}$	$\frac{89}{100}$	$\frac{11}{100}$	$\frac{100}{100}$
12	1	$\frac{12}{100}$	10	10		$\frac{88}{100}$	$\frac{88}{100}$	$\frac{12}{100}$	$\frac{100}{100}$
13	1	$\frac{13}{100}$	10	10		$\frac{87}{100}$	$\frac{87}{100}$	$\frac{13}{100}$	$\frac{100}{100}$
14	1	$\frac{14}{100}$	10	10		$\frac{86}{100}$	$\frac{86}{100}$	$\frac{14}{100}$	$\frac{100}{100}$
15	1	$\frac{15}{100}$	10	10		$\frac{85}{100}$	$\frac{85}{100}$	$\frac{15}{100}$	$\frac{100}{100}$
16	1	$\frac{16}{100}$	10	10		$\frac{84}{100}$	$\frac{84}{100}$	$\frac{16}{100}$	$\frac{100}{100}$
17	1	$\frac{17}{100}$	10	10		$\frac{83}{100}$	$\frac{83}{100}$	$\frac{17}{100}$	$\frac{100}{100}$
18	1	$\frac{18}{100}$	10	10		$\frac{82}{100}$	$\frac{82}{100}$	$\frac{18}{100}$	$\frac{100}{100}$
19	1	$\frac{19}{100}$	10	10		$\frac{81}{100}$	$\frac{81}{100}$	$\frac{19}{100}$	$\frac{100}{100}$
20	1	$\frac{20}{100}$	10	10		$\frac{80}{100}$	$\frac{80}{100}$	$\frac{20}{100}$	$\frac{100}{100}$
21	1	$\frac{21}{100}$	10	10		$\frac{79}{100}$	$\frac{79}{100}$	$\frac{21}{100}$	$\frac{100}{100}$
22	1	$\frac{22}{100}$	10	10		$\frac{78}{100}$	$\frac{78}{100}$	$\frac{22}{100}$	$\frac{100}{100}$
23	1	$\frac{23}{100}$	10	10		$\frac{77}{100}$	$\frac{77}{100}$	$\frac{23}{100}$	$\frac{100}{100}$

No. of the Case.	Weight of A in pounds.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	1	$\frac{24}{100}$	10	10		$\frac{76}{100}$	$\frac{76}{100}$	$\frac{24}{100}$	$\frac{100}{100}$
25	1	$\frac{25}{100}$	10	10		$\frac{75}{100}$	$\frac{75}{100}$	$\frac{25}{100}$	$\frac{100}{100}$
26	1	$\frac{26}{100}$	10	10		$\frac{74}{100}$	$\frac{74}{100}$	$\frac{26}{100}$	$\frac{100}{100}$
27	1	$\frac{27}{100}$	10	10		$\frac{73}{100}$	$\frac{73}{100}$	$\frac{27}{100}$	$\frac{100}{100}$
28	1	$\frac{28}{100}$	10	10		$\frac{72}{100}$	$\frac{72}{100}$	$\frac{28}{100}$	$\frac{100}{100}$
29	1	$\frac{29}{100}$	10	10		$\frac{71}{100}$	$\frac{71}{100}$	$\frac{29}{100}$	$\frac{100}{100}$
30	1	$\frac{30}{100}$	10	10		$\frac{70}{100}$	$\frac{70}{100}$	$\frac{30}{100}$	$\frac{100}{100}$
31	1	$\frac{31}{100}$	10	10		$\frac{69}{100}$	$\frac{69}{100}$	$\frac{31}{100}$	$\frac{100}{100}$
32	1	$\frac{32}{100}$	10	10		$\frac{68}{100}$	$\frac{68}{100}$	$\frac{32}{100}$	$\frac{100}{100}$
33	1	$\frac{33}{100}$	10	10		$\frac{67}{100}$	$\frac{67}{100}$	$\frac{33}{100}$	$\frac{100}{100}$
34	1	$\frac{34}{100}$	10	10		$\frac{66}{100}$	$\frac{66}{100}$	$\frac{34}{100}$	$\frac{100}{100}$
35	1	$\frac{35}{100}$	10	10		$\frac{65}{100}$	$\frac{35}{100}$	$\frac{35}{100}$	$\frac{100}{100}$
36	1	$\frac{36}{100}$	10	10		$\frac{64}{100}$	$\frac{64}{100}$	$\frac{36}{100}$	$\frac{100}{100}$
37	1	$\frac{37}{100}$	10	10		$\frac{23}{100}$	$\frac{68}{100}$	$\frac{37}{100}$	$\frac{100}{100}$
38	1	$\frac{38}{100}$	10	10		$\frac{62}{100}$	$\frac{62}{100}$	$\frac{38}{100}$	$\frac{100}{100}$
39	1	$\frac{39}{100}$	10	10		$\frac{61}{100}$	$\frac{61}{100}$	$\frac{38}{100}$	$\frac{100}{100}$
40	1	$\frac{40}{100}$	10	10		$\frac{60}{100}$	$\frac{60}{100}$	$\frac{40}{100}$	$\frac{100}{100}$
41	1	$\frac{41}{100}$	10	10		$\frac{59}{100}$	$\frac{59}{100}$	$\frac{41}{100}$	$\frac{100}{100}$
42	1	$\frac{42}{100}$	10	10		$\frac{59}{100}$	$\frac{58}{100}$	$\frac{42}{100}$	$\frac{100}{100}$
43	1	$\frac{43}{100}$	10	10		$\frac{57}{100}$	$\frac{57}{100}$	$\frac{43}{100}$	$\frac{100}{100}$
44	1	$\frac{44}{100}$	10	10		$\frac{66}{100}$	$\frac{36}{100}$	$\frac{44}{100}$	$\frac{100}{100}$
45	1	$\frac{45}{100}$	10	10		$\frac{55}{100}$	$\frac{55}{100}$	$\frac{45}{100}$	$\frac{100}{100}$
46	1	$\frac{46}{100}$	10	10		$\frac{54}{100}$	$\frac{54}{100}$	$\frac{46}{100}$	$\frac{100}{100}$
47	1	$\frac{47}{100}$	10	10		$\frac{53}{100}$	$\frac{53}{100}$	$\frac{47}{100}$	$\frac{100}{100}$
48	1	$\frac{48}{100}$	10	10		$\frac{52}{100}$	$\frac{52}{100}$	$\frac{46}{100}$	$\frac{100}{100}$
49	1	$\frac{49}{100}$	10	10		$\frac{51}{100}$	$\frac{51}{100}$	$\frac{49}{100}$	$\frac{100}{100}$

No. of the Case.	Weight of A in pounds.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	1	$\frac{50}{100}$	10	10		$\frac{50}{100}$	$\frac{50}{100}$	$\frac{50}{100}$	$\frac{100}{100}$
51	1	$\frac{51}{100}$	10	10		$\frac{49}{100}$	$\frac{49}{100}$	$\frac{51}{100}$	$\frac{100}{100}$
52	1	$\frac{52}{100}$	10	10		$\frac{48}{100}$	$\frac{49}{100}$	$\frac{52}{100}$	$\frac{100}{100}$
53	1	$\frac{53}{100}$	10	10		$\frac{47}{100}$	$\frac{47}{100}$	$\frac{53}{100}$	$\frac{100}{100}$
54	1	$\frac{54}{100}$	10	10		$\frac{46}{100}$	$\frac{46}{100}$	$\frac{54}{100}$	$\frac{100}{100}$
55	1	$\frac{55}{100}$	10	10		$\frac{45}{100}$	$\frac{45}{100}$	$\frac{55}{100}$	$\frac{100}{100}$
56	1	$\frac{56}{100}$	10	10		$\frac{44}{100}$	$\frac{44}{100}$	$\frac{56}{100}$	$\frac{100}{100}$
57	1	$\frac{57}{100}$	10	10		$\frac{43}{100}$	$\frac{43}{100}$	$\frac{57}{100}$	$\frac{100}{100}$
58	1	$\frac{58}{100}$	10	10		$\frac{42}{100}$	$\frac{42}{100}$	$\frac{58}{100}$	$\frac{100}{100}$
59	1	$\frac{59}{100}$	10	10		$\frac{41}{100}$	$\frac{41}{100}$	$\frac{59}{100}$	$\frac{100}{100}$
60	1	$\frac{60}{100}$	10	10		$\frac{40}{100}$	$\frac{40}{100}$	$\frac{60}{100}$	$\frac{100}{100}$
61	1	$\frac{61}{100}$	10	10		$\frac{39}{100}$	$\frac{39}{100}$	$\frac{61}{100}$	$\frac{100}{100}$
62	1	$\frac{62}{100}$	10	10		$\frac{38}{100}$	$\frac{38}{100}$	$\frac{62}{100}$	$\frac{100}{100}$
63	1	$\frac{63}{100}$	10	10		$\frac{37}{100}$	$\frac{37}{100}$	$\frac{63}{100}$	$\frac{100}{100}$
64	1	$\frac{64}{100}$	10	10		$\frac{36}{100}$	$\frac{36}{100}$	$\frac{64}{100}$	$\frac{100}{100}$
65	1	$\frac{65}{100}$	10	10		$\frac{35}{100}$	$\frac{35}{100}$	$\frac{65}{100}$	$\frac{100}{100}$
66	1	$\frac{66}{100}$	10	10		$\frac{34}{100}$	$\frac{34}{100}$	$\frac{66}{100}$	$\frac{100}{100}$
67	1	$\frac{67}{100}$	10	10		$\frac{33}{100}$	$\frac{33}{100}$	$\frac{67}{100}$	$\frac{100}{100}$
68	1	$\frac{68}{100}$	10	10		$\frac{32}{100}$	$\frac{32}{100}$	$\frac{68}{100}$	$\frac{100}{100}$
69	1	$\frac{69}{100}$	10	10		$\frac{31}{100}$	$\frac{31}{100}$	$\frac{69}{100}$	$\frac{100}{100}$
70	1	$\frac{70}{100}$	10	10		$\frac{30}{100}$	$\frac{30}{100}$	$\frac{70}{100}$	$\frac{100}{100}$
71	1	$\frac{71}{100}$	10	10		$\frac{29}{100}$	$\frac{29}{100}$	$\frac{71}{100}$	$\frac{100}{100}$
72	1	$\frac{72}{100}$	10	10		$\frac{28}{100}$	$\frac{28}{100}$	$\frac{72}{100}$	$\frac{100}{100}$
73	1	$\frac{73}{100}$	10	10		$\frac{27}{100}$	$\frac{27}{100}$	$\frac{73}{100}$	$\frac{100}{100}$
74	1	$\frac{74}{100}$	10	10		$\frac{26}{100}$	$\frac{26}{100}$	$\frac{74}{100}$	$\frac{100}{100}$
75	1	$\frac{75}{100}$	10	10		$\frac{25}{100}$	$\frac{25}{100}$	$\frac{75}{100}$	$\frac{100}{100}$

No. of the Case.	Weight of A in pounds.	Weight of B in fractions of 1 pound.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	1	$\frac{76}{100}$	10	10		$\frac{24}{100}$	$\frac{24}{100}$	$\frac{76}{100}$	$\frac{100}{100}$
77	1	$\frac{77}{100}$	10	10		$\frac{23}{100}$	$\frac{23}{100}$	$\frac{77}{100}$	$\frac{100}{100}$
78	1	$\frac{78}{100}$	10	10		$\frac{22}{100}$	$\frac{22}{100}$	$\frac{78}{100}$	$\frac{100}{100}$
79	1	$\frac{79}{100}$	10	10		$\frac{21}{100}$	$\frac{21}{100}$	$\frac{79}{100}$	$\frac{100}{100}$
80	1	$\frac{80}{100}$	10	10		$\frac{20}{100}$	$\frac{20}{100}$	$\frac{80}{100}$	$\frac{100}{100}$
81	1	$\frac{81}{100}$	10	10		$\frac{19}{100}$	$\frac{19}{100}$	$\frac{81}{100}$	$\frac{100}{100}$
82	1	$\frac{82}{100}$	10	10		$\frac{18}{100}$	$\frac{18}{100}$	$\frac{82}{100}$	$\frac{100}{100}$
83	1	$\frac{83}{100}$	10	10		$\frac{17}{100}$	$\frac{17}{100}$	$\frac{83}{100}$	$\frac{100}{100}$
84	1	$\frac{84}{100}$	10	10		$\frac{16}{100}$	$\frac{16}{100}$	$\frac{84}{100}$	$\frac{100}{100}$
85	1	$\frac{85}{100}$	10	10		$\frac{15}{100}$	$\frac{15}{100}$	$\frac{85}{100}$	$\frac{100}{100}$
86	1	$\frac{86}{100}$	10	10		$\frac{14}{100}$	$\frac{14}{100}$	$\frac{86}{100}$	$\frac{100}{100}$
87	1	$\frac{87}{100}$	10	10		$\frac{13}{100}$	$\frac{13}{100}$	$\frac{87}{100}$	$\frac{100}{100}$
88	1	$\frac{88}{100}$	10	10		$\frac{12}{100}$	$\frac{12}{100}$	$\frac{88}{100}$	$\frac{100}{100}$
89	1	$\frac{89}{100}$	10	10		$\frac{11}{100}$	$\frac{11}{100}$	$\frac{89}{100}$	$\frac{100}{100}$
90	1	$\frac{90}{100}$	10	10		$\frac{10}{100}$	$\frac{10}{100}$	$\frac{90}{100}$	$\frac{100}{100}$
91	1	$\frac{91}{100}$	10	10		$\frac{9}{100}$	$\frac{9}{100}$	$\frac{91}{100}$	$\frac{100}{100}$
92	1	$\frac{92}{100}$	10	10		$\frac{8}{100}$	$\frac{8}{100}$	$\frac{92}{100}$	$\frac{100}{100}$
93	1	$\frac{93}{100}$	10	10		$\frac{7}{100}$	$\frac{7}{100}$	$\frac{93}{100}$	$\frac{100}{100}$
94	1	$\frac{94}{100}$	10	10		$\frac{6}{100}$	$\frac{6}{100}$	$\frac{94}{100}$	$\frac{100}{100}$
95	1	$\frac{95}{100}$	10	10		$\frac{5}{100}$	$\frac{5}{100}$	$\frac{95}{100}$	$\frac{100}{100}$
96	1	$\frac{96}{100}$	10	10		$\frac{4}{100}$	$\frac{4}{100}$	$\frac{96}{100}$	$\frac{100}{100}$
97	1	$\frac{97}{100}$	10	10		$\frac{3}{100}$	$\frac{3}{100}$	$\frac{97}{100}$	$\frac{100}{100}$
98	1	$\frac{98}{100}$	10	10		$\frac{2}{100}$	$\frac{2}{100}$	$\frac{98}{100}$	$\frac{100}{100}$
99	1	$\frac{99}{100}$	10	10		$\frac{1}{100}$	$\frac{1}{100}$	$\frac{99}{100}$	$\frac{100}{100}$
100	1	$\frac{100}{100}$	10	10		$\frac{0}{100}$	$\frac{0}{100}$	$\frac{100}{100}$	$\frac{100}{100}$

TABLE II.

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	$\frac{101}{100}$	1	10	$10\frac{1}{10}$		$\frac{1}{101}$	$\frac{1}{101}$	$\frac{100}{101}$	$\frac{101}{101}$
2	$\frac{102}{100}$	1	10	$10\frac{2}{10}$		$\frac{2}{102}$	$\frac{2}{102}$	$\frac{100}{102}$	$\frac{102}{102}$
3	$\frac{103}{100}$	1	10	$10\frac{3}{10}$		$\frac{3}{103}$	$\frac{3}{103}$	$\frac{100}{103}$	$\frac{103}{103}$
4	$\frac{104}{100}$	1	10	$10\frac{4}{10}$		$\frac{4}{104}$	$\frac{4}{104}$	$\frac{100}{104}$	$\frac{104}{104}$
5	$\frac{105}{100}$	1	10	$10\frac{5}{10}$		$\frac{5}{105}$	$\frac{5}{105}$	$\frac{100}{105}$	$\frac{105}{105}$
6	$\frac{106}{100}$	1	10	$10\frac{6}{10}$		$\frac{6}{106}$	$\frac{6}{106}$	$\frac{100}{106}$	$\frac{106}{106}$
7	$\frac{107}{100}$	1	10	$10\frac{7}{10}$		$\frac{7}{107}$	$\frac{7}{107}$	$\frac{100}{107}$	$\frac{107}{107}$
8	$\frac{108}{100}$	1	10	$10\frac{8}{10}$		$\frac{8}{108}$	$\frac{8}{108}$	$\frac{100}{108}$	$\frac{108}{108}$
9	$\frac{109}{100}$	1	10	$10\frac{9}{10}$		$\frac{9}{109}$	$\frac{9}{109}$	$\frac{100}{109}$	$\frac{109}{109}$
10	$\frac{110}{100}$	1	10	$11\frac{0}{10}$		$\frac{10}{110}$	$\frac{10}{110}$	$\frac{100}{110}$	$\frac{110}{110}$
11	$\frac{111}{100}$	1	10	$11\frac{1}{10}$		$\frac{11}{111}$	$\frac{11}{111}$	$\frac{100}{111}$	$\frac{111}{111}$
12	$\frac{112}{100}$	1	10	$11\frac{2}{10}$		$\frac{12}{112}$	$\frac{12}{112}$	$\frac{100}{112}$	$\frac{112}{112}$
13	$\frac{113}{100}$	1	10	$11\frac{3}{10}$		$\frac{13}{113}$	$\frac{13}{113}$	$\frac{100}{113}$	$\frac{113}{113}$
14	$\frac{114}{100}$	1	10	$11\frac{4}{10}$		$\frac{14}{114}$	$\frac{14}{114}$	$\frac{100}{114}$	$\frac{114}{114}$
15	$\frac{115}{100}$	1	10	$11\frac{5}{10}$		$\frac{15}{115}$	$\frac{15}{115}$	$\frac{100}{115}$	$\frac{115}{115}$
16	$\frac{116}{100}$	1	10	$11\frac{6}{10}$		$\frac{16}{116}$	$\frac{16}{116}$	$\frac{100}{116}$	$\frac{116}{116}$
17	$\frac{117}{100}$	1	10	$11\frac{7}{10}$		$\frac{17}{117}$	$\frac{17}{117}$	$\frac{100}{117}$	$\frac{117}{117}$
18	$\frac{118}{100}$	1	10	$11\frac{8}{10}$		$\frac{18}{118}$	$\frac{18}{118}$	$\frac{100}{118}$	$\frac{118}{118}$
19	$\frac{119}{100}$	1	10	$11\frac{9}{10}$		$\frac{19}{119}$	$\frac{19}{119}$	$\frac{100}{119}$	$\frac{119}{119}$
20	$\frac{120}{100}$	1	10	$12\frac{0}{10}$		$\frac{20}{120}$	$\frac{20}{120}$	$\frac{100}{120}$	$\frac{120}{120}$
21	$\frac{121}{100}$	1	10	$12\frac{1}{10}$		$\frac{21}{121}$	$\frac{21}{121}$	$\frac{100}{121}$	$\frac{121}{121}$
22	$\frac{122}{100}$	1	10	$12\frac{2}{10}$		$\frac{22}{122}$	$\frac{22}{122}$	$\frac{100}{122}$	$\frac{122}{122}$
23	$\frac{123}{100}$	1	10	$12\frac{3}{10}$		$\frac{23}{123}$	$\frac{23}{123}$	$\frac{100}{123}$	$\frac{123}{123}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	$\frac{124}{100}$	1	10	$12\frac{4}{10}$		$\frac{24}{124}$	$\frac{24}{124}$	$\frac{100}{124}$	$\frac{124}{124}$
25	$\frac{125}{100}$	1	10	$12\frac{5}{10}$		$\frac{25}{125}$	$\frac{25}{125}$	$\frac{100}{125}$	$\frac{125}{125}$
26	$\frac{126}{100}$	1	10	$12\frac{6}{10}$		$\frac{26}{126}$	$\frac{26}{126}$	$\frac{100}{126}$	$\frac{126}{126}$
27	$\frac{127}{100}$	1	10	$12\frac{7}{10}$		$\frac{27}{127}$	$\frac{27}{127}$	$\frac{100}{127}$	$\frac{127}{127}$
28	$\frac{128}{100}$	1	10	$12\frac{8}{10}$		$\frac{28}{128}$	$\frac{28}{128}$	$\frac{100}{128}$	$\frac{128}{128}$
29	$\frac{129}{100}$	1	10	$12\frac{9}{10}$		$\frac{29}{129}$	$\frac{29}{129}$	$\frac{100}{129}$	$\frac{129}{129}$
30	$\frac{130}{100}$	1	10	$13\frac{0}{10}$		$\frac{30}{130}$	$\frac{30}{130}$	$\frac{100}{130}$	$\frac{130}{130}$
31	$\frac{131}{100}$	1	10	$13\frac{1}{10}$		$\frac{31}{131}$	$\frac{31}{131}$	$\frac{100}{131}$	$\frac{131}{131}$
32	$\frac{132}{100}$	1	10	$13\frac{2}{10}$		$\frac{32}{132}$	$\frac{32}{132}$	$\frac{100}{132}$	$\frac{132}{132}$
33	$\frac{133}{100}$	1	10	$13\frac{3}{10}$		$\frac{33}{133}$	$\frac{33}{133}$	$\frac{100}{133}$	$\frac{133}{133}$
34	$\frac{134}{100}$	1	10	$13\frac{4}{10}$		$\frac{34}{134}$	$\frac{34}{134}$	$\frac{100}{134}$	$\frac{134}{134}$
35	$\frac{135}{100}$	1	10	$13\frac{5}{10}$		$\frac{35}{135}$	$\frac{35}{135}$	$\frac{100}{135}$	$\frac{135}{135}$
36	$\frac{136}{100}$	1	10	$13\frac{6}{10}$		$\frac{36}{136}$	$\frac{36}{136}$	$\frac{100}{136}$	$\frac{136}{136}$
37	$\frac{137}{100}$	1	10	$13\frac{7}{10}$		$\frac{37}{137}$	$\frac{37}{137}$	$\frac{100}{137}$	$\frac{137}{137}$
38	$\frac{138}{100}$	1	10	$13\frac{8}{10}$		$\frac{38}{138}$	$\frac{38}{138}$	$\frac{100}{138}$	$\frac{138}{138}$
39	$\frac{139}{100}$	1	10	$13\frac{9}{10}$		$\frac{39}{139}$	$\frac{39}{139}$	$\frac{100}{139}$	$\frac{139}{139}$
40	$\frac{140}{100}$	1	10	$14\frac{0}{10}$		$\frac{40}{140}$	$\frac{40}{140}$	$\frac{100}{140}$	$\frac{140}{140}$
41	$\frac{141}{100}$	1	10	$14\frac{1}{10}$		$\frac{41}{141}$	$\frac{41}{141}$	$\frac{100}{141}$	$\frac{141}{141}$
42	$\frac{142}{100}$	1	10	$14\frac{2}{10}$		$\frac{42}{142}$	$\frac{42}{142}$	$\frac{100}{142}$	$\frac{142}{142}$
43	$\frac{143}{100}$	1	10	$14\frac{3}{10}$		$\frac{43}{143}$	$\frac{43}{143}$	$\frac{100}{143}$	$\frac{143}{143}$
44	$\frac{144}{100}$	1	10	$14\frac{4}{10}$		$\frac{44}{144}$	$\frac{44}{144}$	$\frac{100}{144}$	$\frac{144}{144}$
45	$\frac{145}{100}$	1	10	$14\frac{5}{10}$		$\frac{45}{145}$	$\frac{45}{145}$	$\frac{100}{145}$	$\frac{145}{145}$
46	$\frac{146}{100}$	1	10	$14\frac{6}{10}$		$\frac{46}{146}$	$\frac{46}{146}$	$\frac{100}{146}$	$\frac{146}{146}$
47	$\frac{147}{100}$	1	10	$14\frac{7}{10}$		$\frac{47}{147}$	$\frac{47}{147}$	$\frac{100}{147}$	$\frac{147}{147}$
48	$\frac{148}{100}$	1	10	$14\frac{8}{10}$		$\frac{48}{148}$	$\frac{48}{148}$	$\frac{100}{148}$	$\frac{148}{148}$
49	$\frac{149}{100}$	1	10	$14\frac{9}{10}$		$\frac{49}{149}$	$\frac{49}{149}$	$\frac{100}{149}$	$\frac{149}{149}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	$\frac{150}{100}$	1	10	$15\frac{0}{10}$		$\frac{50}{150}$	$\frac{50}{150}$	$\frac{100}{150}$	$\frac{150}{150}$
51	$\frac{151}{100}$	1	10	$15\frac{1}{10}$		$\frac{51}{151}$	$\frac{51}{151}$	$\frac{100}{151}$	$\frac{151}{151}$
52	$\frac{152}{100}$	1	10	$15\frac{2}{10}$		$\frac{52}{152}$	$\frac{52}{152}$	$\frac{100}{152}$	$\frac{152}{152}$
53	$\frac{153}{100}$	1	10	$15\frac{3}{10}$		$\frac{53}{153}$	$\frac{53}{153}$	$\frac{100}{153}$	$\frac{153}{153}$
54	$\frac{154}{100}$	1	10	$15\frac{4}{10}$		$\frac{54}{154}$	$\frac{54}{154}$	$\frac{100}{154}$	$\frac{154}{154}$
55	$\frac{155}{100}$	1	10	$15\frac{5}{10}$		$\frac{55}{155}$	$\frac{55}{155}$	$\frac{100}{155}$	$\frac{155}{155}$
56	$\frac{156}{100}$	1	10	$15\frac{6}{10}$		$\frac{56}{156}$	$\frac{56}{156}$	$\frac{100}{156}$	$\frac{156}{156}$
57	$\frac{157}{100}$	1	10	$15\frac{7}{10}$		$\frac{57}{157}$	$\frac{57}{157}$	$\frac{100}{157}$	$\frac{157}{157}$
58	$\frac{158}{100}$	1	10	$15\frac{8}{10}$		$\frac{58}{158}$	$\frac{58}{158}$	$\frac{100}{158}$	$\frac{158}{158}$
59	$\frac{159}{100}$	1	10	$15\frac{9}{10}$		$\frac{59}{159}$	$\frac{59}{159}$	$\frac{100}{159}$	$\frac{159}{159}$
60	$\frac{160}{100}$	1	10	$16\frac{0}{10}$		$\frac{60}{160}$	$\frac{60}{160}$	$\frac{100}{160}$	$\frac{160}{160}$
61	$\frac{161}{100}$	1	10	$16\frac{1}{10}$		$\frac{61}{161}$	$\frac{61}{161}$	$\frac{100}{161}$	$\frac{161}{161}$
62	$\frac{162}{100}$	1	10	$16\frac{2}{10}$		$\frac{62}{162}$	$\frac{62}{162}$	$\frac{100}{162}$	$\frac{162}{162}$
63	$\frac{163}{100}$	1	10	$16\frac{3}{10}$		$\frac{63}{163}$	$\frac{63}{163}$	$\frac{100}{163}$	$\frac{163}{163}$
64	$\frac{164}{100}$	1	10	$16\frac{4}{10}$		$\frac{64}{164}$	$\frac{64}{164}$	$\frac{100}{164}$	$\frac{164}{164}$
65	$\frac{165}{100}$	1	10	$16\frac{5}{10}$		$\frac{65}{165}$	$\frac{65}{165}$	$\frac{100}{165}$	$\frac{165}{165}$
66	$\frac{166}{100}$	1	10	$16\frac{6}{10}$		$\frac{66}{166}$	$\frac{66}{166}$	$\frac{100}{166}$	$\frac{166}{166}$
67	$\frac{167}{100}$	1	10	$16\frac{7}{10}$		$\frac{67}{167}$	$\frac{67}{167}$	$\frac{100}{167}$	$\frac{167}{167}$
68	$\frac{168}{100}$	1	10	$16\frac{8}{10}$		$\frac{68}{168}$	$\frac{68}{168}$	$\frac{100}{168}$	$\frac{168}{168}$
69	$\frac{169}{100}$	1	10	$16\frac{9}{10}$		$\frac{69}{169}$	$\frac{69}{169}$	$\frac{100}{169}$	$\frac{169}{169}$
70	$\frac{170}{100}$	1	10	$17\frac{0}{10}$		$\frac{70}{170}$	$\frac{70}{170}$	$\frac{100}{170}$	$\frac{170}{170}$
71	$\frac{171}{100}$	1	10	$17\frac{1}{10}$		$\frac{71}{171}$	$\frac{71}{171}$	$\frac{100}{171}$	$\frac{171}{171}$
72	$\frac{172}{100}$	1	10	$17\frac{2}{10}$		$\frac{72}{172}$	$\frac{72}{172}$	$\frac{100}{172}$	$\frac{172}{172}$
73	$\frac{173}{100}$	1	10	$17\frac{3}{10}$		$\frac{73}{173}$	$\frac{73}{173}$	$\frac{100}{173}$	$\frac{173}{173}$
74	$\frac{174}{100}$	1	10	$17\frac{4}{10}$		$\frac{74}{174}$	$\frac{74}{174}$	$\frac{100}{174}$	$\frac{174}{174}$
75	$\frac{175}{100}$	1	10	$17\frac{5}{10}$		$\frac{75}{175}$	$\frac{75}{175}$	$\frac{100}{175}$	$\frac{175}{175}$

No. of the Case.	Weight of A in fractions of 1 pound.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	$\frac{176}{100}$	1	10	$17\frac{6}{10}$		$\frac{76}{176}$	$\frac{76}{176}$	$\frac{100}{176}$	$\frac{176}{176}$
77	$\frac{177}{100}$	1	10	$17\frac{7}{10}$		$\frac{77}{177}$	$\frac{77}{177}$	$\frac{100}{177}$	$\frac{177}{177}$
78	$\frac{178}{100}$	1	10	$17\frac{8}{10}$		$\frac{78}{178}$	$\frac{78}{178}$	$\frac{100}{178}$	$\frac{178}{178}$
79	$\frac{179}{100}$	1	10	$17\frac{9}{10}$		$\frac{79}{179}$	$\frac{79}{179}$	$\frac{100}{179}$	$\frac{179}{179}$
80	$\frac{180}{100}$	1	10	$18\frac{0}{10}$		$\frac{80}{180}$	$\frac{80}{180}$	$\frac{100}{180}$	$\frac{180}{180}$
81	$\frac{181}{100}$	1	10	$18\frac{1}{10}$		$\frac{81}{181}$	$\frac{81}{181}$	$\frac{100}{181}$	$\frac{181}{181}$
82	$\frac{182}{100}$	1	10	$18\frac{2}{10}$		$\frac{82}{182}$	$\frac{82}{182}$	$\frac{100}{182}$	$\frac{182}{182}$
83	$\frac{183}{100}$	1	10	$18\frac{3}{10}$		$\frac{83}{183}$	$\frac{83}{183}$	$\frac{100}{183}$	$\frac{183}{183}$
84	$\frac{184}{100}$	1	10	$18\frac{4}{10}$		$\frac{84}{184}$	$\frac{84}{184}$	$\frac{100}{184}$	$\frac{184}{184}$
85	$\frac{185}{100}$	1	10	$18\frac{5}{10}$		$\frac{85}{185}$	$\frac{85}{185}$	$\frac{100}{185}$	$\frac{185}{185}$
86	$\frac{186}{100}$	1	10	$18\frac{6}{10}$		$\frac{86}{186}$	$\frac{86}{186}$	$\frac{100}{186}$	$\frac{186}{186}$
87	$\frac{187}{100}$	1	10	$18\frac{7}{10}$		$\frac{87}{187}$	$\frac{87}{187}$	$\frac{100}{187}$	$\frac{187}{187}$
88	$\frac{188}{100}$	1	10	$18\frac{8}{10}$		$\frac{88}{188}$	$\frac{88}{188}$	$\frac{100}{188}$	$\frac{188}{188}$
89	$\frac{189}{100}$	1	10	$18\frac{9}{10}$		$\frac{89}{189}$	$\frac{89}{189}$	$\frac{100}{189}$	$\frac{189}{189}$
90	$\frac{190}{100}$	1	10	$19\frac{0}{10}$		$\frac{90}{190}$	$\frac{90}{190}$	$\frac{100}{190}$	$\frac{190}{190}$
91	$\frac{191}{100}$	1	10	$19\frac{1}{10}$		$\frac{91}{191}$	$\frac{91}{191}$	$\frac{100}{191}$	$\frac{191}{191}$
92	$\frac{192}{100}$	1	10	$19\frac{2}{10}$		$\frac{92}{192}$	$\frac{92}{192}$	$\frac{100}{192}$	$\frac{192}{192}$
93	$\frac{193}{100}$	1	10	$19\frac{3}{10}$		$\frac{93}{193}$	$\frac{93}{193}$	$\frac{100}{193}$	$\frac{193}{193}$
94	$\frac{194}{100}$	1	10	$19\frac{4}{10}$		$\frac{94}{194}$	$\frac{94}{194}$	$\frac{100}{194}$	$\frac{194}{194}$
95	$\frac{195}{100}$	1	10	$19\frac{5}{10}$		$\frac{95}{195}$	$\frac{95}{195}$	$\frac{100}{195}$	$\frac{195}{195}$
96	$\frac{196}{100}$	1	10	$19\frac{6}{10}$		$\frac{96}{196}$	$\frac{96}{196}$	$\frac{100}{196}$	$\frac{196}{196}$
97	$\frac{197}{100}$	1	10	$19\frac{7}{10}$		$\frac{97}{197}$	$\frac{97}{197}$	$\frac{100}{197}$	$\frac{197}{197}$
98	$\frac{198}{100}$	1	10	$19\frac{8}{10}$		$\frac{98}{198}$	$\frac{98}{198}$	$\frac{100}{198}$	$\frac{198}{198}$
99	$\frac{199}{100}$	1	10	$19\frac{9}{10}$		$\frac{99}{199}$	$\frac{99}{199}$	$\frac{100}{199}$	$\frac{199}{199}$
100	$\frac{200}{100}$	1	10	$20\frac{0}{10}$		$\frac{100}{200}$	$\frac{100}{200}$	$\frac{100}{200}$	$\frac{200}{200}$

TABLE III.

SECTION II.] [CLASS II.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	1	1	10	10		$\frac{0}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
2	2	1	10	20		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2}$
3	3	1	10	30		$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$
4	4	1	10	40		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{4}{4}$
5	5	1	10	50		$\frac{4}{5}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{5}{5}$
6	6	1	10	60		$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{6}{6}$
7	7	1	10	70		$\frac{6}{7}$	$\frac{6}{7}$	$\frac{1}{7}$	$\frac{7}{7}$
8	8	1	10	80		$\frac{7}{8}$	$\frac{7}{8}$	$\frac{1}{8}$	$\frac{8}{8}$
9	9	1	10	90		$\frac{8}{9}$	$\frac{8}{9}$	$\frac{1}{9}$	$\frac{9}{9}$
10	10	1	10	100		$\frac{9}{10}$	$\frac{9}{10}$	$\frac{1}{10}$	$\frac{10}{10}$
11	11	1	10	110		$\frac{10}{11}$	$\frac{10}{11}$	$\frac{1}{11}$	$\frac{11}{11}$
12	12	1	10	120		$\frac{11}{12}$	$\frac{11}{12}$	$\frac{1}{12}$	$\frac{12}{12}$
13	13	1	10	130		$\frac{12}{13}$	$\frac{12}{13}$	$\frac{1}{13}$	$\frac{13}{13}$
14	14	1	10	140		$\frac{13}{14}$	$\frac{13}{14}$	$\frac{1}{14}$	$\frac{14}{14}$
15	15	1	10	150		$\frac{14}{15}$	$\frac{14}{15}$	$\frac{1}{15}$	$\frac{15}{15}$
16	16	1	10	160		$\frac{15}{16}$	$\frac{15}{16}$	$\frac{1}{16}$	$\frac{16}{16}$
17	17	1	10	170		$\frac{16}{17}$	$\frac{16}{17}$	$\frac{1}{17}$	$\frac{17}{17}$
18	18	1	10	180		$\frac{17}{18}$	$\frac{17}{18}$	$\frac{1}{18}$	$\frac{18}{18}$
19	19	1	10	190		$\frac{18}{19}$	$\frac{18}{19}$	$\frac{1}{19}$	$\frac{19}{19}$
20	20	1	10	200		$\frac{19}{20}$	$\frac{19}{20}$	$\frac{1}{20}$	$\frac{20}{20}$
21	21	1	10	210		$\frac{20}{21}$	$\frac{20}{21}$	$\frac{1}{21}$	$\frac{21}{21}$
22	22	1	10	220		$\frac{21}{22}$	$\frac{21}{22}$	$\frac{1}{22}$	$\frac{22}{22}$
23	23	1	10	230		$\frac{22}{23}$	$\frac{22}{23}$	$\frac{1}{23}$	$\frac{23}{23}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	24	1	10	240		$\frac{23}{24}$	$\frac{23}{24}$	$\frac{1}{24}$	$\frac{24}{24}$
25	25	1	10	250		$\frac{24}{25}$	$\frac{24}{25}$	$\frac{1}{25}$	$\frac{25}{25}$
26	26	1	10	260		$\frac{25}{26}$	$\frac{25}{26}$	$\frac{1}{26}$	$\frac{26}{26}$
27	27	1	10	270		$\frac{26}{27}$	$\frac{26}{27}$	$\frac{1}{27}$	$\frac{27}{27}$
28	28	1	10	280		$\frac{27}{28}$	$\frac{27}{28}$	$\frac{1}{28}$	$\frac{28}{28}$
29	29	1	10	290		$\frac{28}{29}$	$\frac{28}{29}$	$\frac{1}{29}$	$\frac{29}{29}$
30	30	1	10	300		$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{30}$	$\frac{30}{30}$
31	31	1	10	310		$\frac{30}{31}$	$\frac{30}{31}$	$\frac{1}{31}$	$\frac{31}{31}$
32	32	1	10	320		$\frac{31}{32}$	$\frac{31}{32}$	$\frac{1}{32}$	$\frac{32}{32}$
33	33	1	10	330		$\frac{32}{33}$	$\frac{32}{33}$	$\frac{1}{33}$	$\frac{33}{33}$
34	34	1	10	340		$\frac{33}{34}$	$\frac{33}{34}$	$\frac{1}{34}$	$\frac{34}{34}$
35	35	1	10	350		$\frac{34}{35}$	$\frac{34}{35}$	$\frac{1}{35}$	$\frac{35}{35}$
36	36	1	10	360		$\frac{35}{36}$	$\frac{35}{36}$	$\frac{1}{36}$	$\frac{36}{36}$
37	37	1	10	370		$\frac{36}{37}$	$\frac{36}{37}$	$\frac{1}{37}$	$\frac{37}{37}$
38	38	1	10	380		$\frac{37}{38}$	$\frac{37}{38}$	$\frac{1}{38}$	$\frac{38}{38}$
39	39	1	10	390		$\frac{38}{39}$	$\frac{38}{39}$	$\frac{1}{39}$	$\frac{39}{39}$
40	40	1	10	400		$\frac{39}{40}$	$\frac{39}{40}$	$\frac{1}{40}$	$\frac{40}{40}$
41	41	1	10	410		$\frac{40}{41}$	$\frac{40}{41}$	$\frac{1}{41}$	$\frac{41}{41}$
42	42	1	10	420		$\frac{41}{42}$	$\frac{41}{42}$	$\frac{1}{42}$	$\frac{42}{42}$
43	43	1	10	430		$\frac{42}{43}$	$\frac{42}{43}$	$\frac{1}{43}$	$\frac{43}{43}$
44	44	1	10	440		$\frac{43}{44}$	$\frac{43}{44}$	$\frac{1}{44}$	$\frac{44}{44}$
45	45	1	10	450		$\frac{44}{45}$	$\frac{44}{45}$	$\frac{1}{45}$	$\frac{45}{45}$
46	46	1	10	460		$\frac{45}{46}$	$\frac{45}{46}$	$\frac{1}{46}$	$\frac{46}{46}$
47	47	1	10	470		$\frac{46}{47}$	$\frac{46}{47}$	$\frac{1}{47}$	$\frac{47}{47}$
48	48	1	10	480		$\frac{47}{48}$	$\frac{47}{48}$	$\frac{1}{48}$	$\frac{48}{48}$
49	49	1	10	490		$\frac{48}{49}$	$\frac{48}{49}$	$\frac{1}{49}$	$\frac{49}{49}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	50	1	10	500		$\frac{49}{50}$	$\frac{49}{50}$	$\frac{1}{50}$	$\frac{50}{50}$
51	51	1	10	510		$\frac{50}{51}$	$\frac{50}{51}$	$\frac{1}{51}$	$\frac{51}{51}$
52	52	1	10	520		$\frac{51}{52}$	$\frac{51}{52}$	$\frac{1}{52}$	$\frac{52}{52}$
53	53	1	10	530		$\frac{52}{53}$	$\frac{52}{53}$	$\frac{1}{53}$	$\frac{53}{53}$
54	54	1	10	540		$\frac{53}{54}$	$\frac{53}{54}$	$\frac{1}{54}$	$\frac{54}{54}$
55	55	1	10	550		$\frac{54}{55}$	$\frac{54}{55}$	$\frac{1}{55}$	$\frac{55}{55}$
56	56	1	10	560		$\frac{55}{56}$	$\frac{55}{56}$	$\frac{1}{56}$	$\frac{56}{56}$
57	57	1	10	570		$\frac{56}{57}$	$\frac{56}{57}$	$\frac{1}{57}$	$\frac{57}{57}$
58	58	1	10	580		$\frac{57}{58}$	$\frac{57}{58}$	$\frac{1}{58}$	$\frac{58}{58}$
59	59	1	10	590		$\frac{58}{59}$	$\frac{58}{59}$	$\frac{1}{59}$	$\frac{59}{59}$
60	60	1	10	600		$\frac{59}{60}$	$\frac{59}{60}$	$\frac{1}{60}$	$\frac{60}{60}$
61	61	1	10	610		$\frac{60}{61}$	$\frac{60}{61}$	$\frac{1}{61}$	$\frac{61}{61}$
62	62	1	10	620		$\frac{61}{62}$	$\frac{61}{62}$	$\frac{1}{62}$	$\frac{62}{62}$
63	63	1	10	630		$\frac{62}{63}$	$\frac{62}{63}$	$\frac{1}{63}$	$\frac{63}{63}$
64	64	1	10	640		$\frac{63}{64}$	$\frac{63}{64}$	$\frac{1}{64}$	$\frac{64}{64}$
65	65	1	10	650		$\frac{64}{65}$	$\frac{64}{65}$	$\frac{1}{65}$	$\frac{65}{65}$
66	66	1	10	660		$\frac{65}{66}$	$\frac{65}{66}$	$\frac{1}{66}$	$\frac{66}{66}$
67	67	1	10	670		$\frac{66}{67}$	$\frac{66}{67}$	$\frac{1}{67}$	$\frac{67}{67}$
68	68	1	10	680		$\frac{67}{68}$	$\frac{67}{68}$	$\frac{1}{68}$	$\frac{68}{68}$
69	69	1	10	690		$\frac{68}{69}$	$\frac{68}{69}$	$\frac{1}{69}$	$\frac{69}{69}$
70	70	1	10	700		$\frac{69}{70}$	$\frac{69}{70}$	$\frac{1}{70}$	$\frac{70}{70}$
71	71	1	10	710		$\frac{70}{71}$	$\frac{70}{71}$	$\frac{1}{71}$	$\frac{71}{71}$
72	72	1	10	720		$\frac{71}{72}$	$\frac{71}{72}$	$\frac{1}{72}$	$\frac{72}{72}$
73	73	1	10	730		$\frac{72}{73}$	$\frac{72}{73}$	$\frac{1}{73}$	$\frac{73}{73}$
74	74	1	10	740		$\frac{73}{74}$	$\frac{73}{74}$	$\frac{1}{74}$	$\frac{74}{74}$
75	75	1	10	750		$\frac{74}{75}$	$\frac{74}{75}$	$\frac{1}{75}$	$\frac{75}{75}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	76	1	10	760		$\frac{75}{76}$	$\frac{75}{76}$	$\frac{1}{76}$	$\frac{76}{76}$
77	77	1	10	770		$\frac{76}{77}$	$\frac{76}{77}$	$\frac{1}{77}$	$\frac{77}{77}$
78	78	1	10	780		$\frac{77}{78}$	$\frac{77}{78}$	$\frac{1}{78}$	$\frac{78}{78}$
79	79	1	10	790		$\frac{78}{79}$	$\frac{78}{79}$	$\frac{1}{79}$	$\frac{79}{79}$
80	80	1	10	800		$\frac{79}{80}$	$\frac{79}{80}$	$\frac{1}{80}$	$\frac{80}{80}$
81	81	1	10	810		$\frac{80}{81}$	$\frac{80}{81}$	$\frac{1}{81}$	$\frac{81}{81}$
82	82	1	10	820		$\frac{81}{82}$	$\frac{81}{82}$	$\frac{1}{82}$	$\frac{82}{82}$
83	83	1	10	830		$\frac{82}{83}$	$\frac{82}{83}$	$\frac{1}{83}$	$\frac{83}{83}$
84	84	1	10	840		$\frac{83}{84}$	$\frac{83}{84}$	$\frac{1}{84}$	$\frac{84}{84}$
85	85	1	10	850		$\frac{84}{85}$	$\frac{84}{85}$	$\frac{1}{85}$	$\frac{85}{85}$
86	86	1	10	860		$\frac{85}{86}$	$\frac{85}{86}$	$\frac{1}{86}$	$\frac{86}{86}$
87	87	1	10	870		$\frac{86}{87}$	$\frac{86}{87}$	$\frac{1}{87}$	$\frac{87}{87}$
88	88	1	10	880		$\frac{87}{88}$	$\frac{87}{88}$	$\frac{1}{88}$	$\frac{88}{88}$
89	89	1	10	890		$\frac{88}{89}$	$\frac{88}{89}$	$\frac{1}{89}$	$\frac{89}{89}$
90	90	1	10	900		$\frac{89}{90}$	$\frac{89}{90}$	$\frac{1}{90}$	$\frac{90}{90}$
91	91	1	10	910		$\frac{90}{91}$	$\frac{90}{91}$	$\frac{1}{91}$	$\frac{91}{91}$
92	92	1	10	920		$\frac{91}{92}$	$\frac{91}{92}$	$\frac{1}{92}$	$\frac{92}{92}$
93	93	1	10	930		$\frac{92}{93}$	$\frac{92}{93}$	$\frac{1}{93}$	$\frac{93}{93}$
94	94	1	10	940		$\frac{93}{94}$	$\frac{93}{94}$	$\frac{1}{94}$	$\frac{94}{94}$
95	95	1	10	950		$\frac{94}{95}$	$\frac{94}{95}$	$\frac{1}{95}$	$\frac{95}{95}$
96	96	1	10	960		$\frac{95}{96}$	$\frac{95}{96}$	$\frac{1}{96}$	$\frac{96}{96}$
97	97	1	10	970		$\frac{96}{97}$	$\frac{96}{97}$	$\frac{1}{97}$	$\frac{97}{97}$
98	98	1	10	980		$\frac{97}{98}$	$\frac{97}{98}$	$\frac{1}{98}$	$\frac{98}{98}$
99	99	1	10	990		$\frac{98}{99}$	$\frac{98}{99}$	$\frac{1}{99}$	$\frac{99}{99}$
100	100	1	10	1000		$\frac{99}{100}$	$\frac{99}{100}$	$\frac{1}{100}$	$\frac{100}{100}$

TABLE IV.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	3	2	10	30		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
2	4	2	10	40		$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
3	5	2	10	50		$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{5}{5}$
4	6	2	10	60		$\frac{4}{6}$	$\frac{4}{6}$	$\frac{2}{6}$	$\frac{6}{6}$
5	7	2	10	70		$\frac{5}{7}$	$\frac{5}{7}$	$\frac{2}{7}$	$\frac{7}{7}$
6	8	2	10	80		$\frac{6}{8}$	$\frac{6}{8}$	$\frac{2}{8}$	$\frac{8}{8}$
7	9	2	10	90		$\frac{7}{9}$	$\frac{7}{9}$	$\frac{2}{9}$	$\frac{9}{9}$
8	10	2	10	100		$\frac{8}{10}$	$\frac{8}{10}$	$\frac{2}{10}$	$\frac{10}{10}$
9	11	2	10	110		$\frac{9}{11}$	$\frac{9}{11}$	$\frac{2}{11}$	$\frac{11}{11}$
10	12	2	10	120		$\frac{10}{12}$	$\frac{10}{12}$	$\frac{2}{12}$	$\frac{12}{12}$
11	13	2	10	130		$\frac{11}{13}$	$\frac{11}{13}$	$\frac{2}{13}$	$\frac{13}{13}$
12	14	2	10	140		$\frac{12}{14}$	$\frac{12}{14}$	$\frac{2}{14}$	$\frac{14}{14}$
13	15	2	10	150		$\frac{13}{15}$	$\frac{13}{15}$	$\frac{2}{15}$	$\frac{15}{15}$
14	16	2	10	160		$\frac{14}{16}$	$\frac{14}{16}$	$\frac{2}{16}$	$\frac{16}{16}$
15	4	3	10	40		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
16	5	3	10	50		$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{5}{5}$
17	6	3	10	60		$\frac{3}{6}$	$\frac{3}{6}$	$\frac{3}{6}$	$\frac{6}{6}$
18	7	3	10	70		$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{7}{7}$
19	8	3	10	80		$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{8}{8}$
20	9	3	10	90		$\frac{6}{9}$	$\frac{6}{9}$	$\frac{3}{9}$	$\frac{9}{9}$
21	10	3	10	100		$\frac{7}{10}$	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{10}{10}$
22	11	3	10	110		$\frac{8}{11}$	$\frac{8}{11}$	$\frac{3}{11}$	$\frac{11}{11}$
23	12	3	10	120		$\frac{9}{12}$	$\frac{9}{12}$	$\frac{3}{12}$	$\frac{12}{12}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
24	13	3	10	130		$\frac{10}{13}$	$\frac{10}{13}$	$\frac{3}{13}$	$\frac{13}{13}$
25	14	3	10	140		$\frac{11}{14}$	$\frac{11}{14}$	$\frac{3}{14}$	$\frac{14}{14}$
26	15	3	10	150		$\frac{12}{15}$	$\frac{12}{15}$	$\frac{3}{15}$	$\frac{15}{15}$
27	16	3	10	160		$\frac{13}{16}$	$\frac{13}{16}$	$\frac{3}{16}$	$\frac{16}{16}$
28	5	4	10	50		$\frac{1}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
29	6	4	10	60		$\frac{2}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{6}{6}$
30	7	4	10	70		$\frac{3}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{7}{7}$
31	8	4	10	80		$\frac{4}{8}$	$\frac{4}{8}$	$\frac{4}{8}$	$\frac{8}{8}$
32	9	4	10	90		$\frac{5}{9}$	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{9}{9}$
33	10	4	10	100		$\frac{6}{10}$	$\frac{6}{10}$	$\frac{4}{10}$	$\frac{10}{10}$
34	11	4	10	110		$\frac{7}{11}$	$\frac{7}{11}$	$\frac{4}{11}$	$\frac{11}{11}$
35	12	4	10	120		$\frac{8}{12}$	$\frac{8}{12}$	$\frac{4}{12}$	$\frac{12}{12}$
36	13	4	10	130		$\frac{9}{13}$	$\frac{9}{13}$	$\frac{4}{13}$	$\frac{13}{13}$
37	14	4	10	140		$\frac{10}{14}$	$\frac{10}{14}$	$\frac{4}{14}$	$\frac{14}{14}$
38	15	4	10	150		$\frac{11}{15}$	$\frac{11}{15}$	$\frac{4}{15}$	$\frac{15}{15}$
39	16	4	10	160		$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	$\frac{16}{16}$
40	6	5	10	60		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{6}{6}$
41	7	5	10	70		$\frac{2}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{7}{7}$
42	8	5	10	80		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{8}{8}$
43	9	5	10	90		$\frac{4}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{9}{9}$
44	10	5	10	100		$\frac{5}{10}$	$\frac{5}{10}$	$\frac{5}{10}$	$\frac{10}{10}$
45	11	5	10	110		$\frac{6}{11}$	$\frac{6}{11}$	$\frac{5}{11}$	$\frac{11}{11}$
46	12	5	10	120		$\frac{7}{12}$	$\frac{7}{12}$	$\frac{5}{12}$	$\frac{12}{12}$
47	13	5	10	130		$\frac{8}{13}$	$\frac{8}{13}$	$\frac{5}{13}$	$\frac{13}{13}$
48	14	5	10	140		$\frac{9}{14}$	$\frac{9}{14}$	$\frac{5}{14}$	$\frac{14}{14}$
49	15	5	10	150		$\frac{10}{15}$	$\frac{10}{15}$	$\frac{5}{15}$	$\frac{15}{15}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
50	16	5	10	160		$\frac{11}{16}$	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{16}{16}$
51	7	6	10	70		$\frac{1}{7}$	$\frac{1}{7}$	$\frac{6}{7}$	$\frac{7}{7}$
52	8	6	10	80		$\frac{2}{8}$	$\frac{2}{8}$	$\frac{6}{8}$	$\frac{8}{8}$
53	9	6	10	90		$\frac{3}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{9}{9}$
54	10	6	10	100		$\frac{4}{10}$	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{10}{10}$
55	11	6	10	110		$\frac{5}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{11}{11}$
56	12	6	10	120		$\frac{6}{12}$	$\frac{6}{12}$	$\frac{6}{12}$	$\frac{12}{12}$
57	13	6	10	130		$\frac{7}{13}$	$\frac{7}{13}$	$\frac{6}{13}$	$\frac{13}{13}$
58	14	6	10	140		$\frac{6}{14}$	$\frac{8}{14}$	$\frac{6}{14}$	$\frac{14}{14}$
59	15	6	10	150		$\frac{9}{15}$	$\frac{9}{15}$	$\frac{6}{15}$	$\frac{15}{15}$
60	16	6	10	160		$\frac{10}{16}$	$\frac{10}{16}$	$\frac{6}{16}$	$\frac{16}{16}$
61	8	7	10	80		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
62	9	7	10	90		$\frac{2}{9}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{9}{9}$
63	10	7	10	100		$\frac{3}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{10}{10}$
64	11	7	10	110		$\frac{4}{11}$	$\frac{4}{11}$	$\frac{7}{11}$	$\frac{11}{11}$
65	12	7	10	120		$\frac{5}{12}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{12}{12}$
66	13	7	10	130		$\frac{6}{13}$	$\frac{6}{13}$	$\frac{7}{13}$	$\frac{13}{13}$
67	14	7	10	140		$\frac{7}{14}$	$\frac{7}{14}$	$\frac{7}{14}$	$\frac{14}{14}$
68	15	7	10	150		$\frac{8}{15}$	$\frac{8}{15}$	$\frac{7}{15}$	$\frac{15}{15}$
69	16	7	10	160		$\frac{9}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	$\frac{16}{16}$
70	9	8	10	90		$\frac{1}{9}$	$\frac{1}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
71	10	8	10	100		$\frac{2}{10}$	$\frac{2}{10}$	$\frac{8}{10}$	$\frac{10}{10}$
72	11	8	10	110		$\frac{3}{11}$	$\frac{3}{11}$	$\frac{8}{11}$	$\frac{11}{11}$
73	12	8	10	120		$\frac{4}{12}$	$\frac{4}{12}$	$\frac{6}{12}$	$\frac{12}{12}$
74	13	8	10	130		$\frac{5}{13}$	$\frac{5}{13}$	$\frac{8}{13}$	$\frac{13}{13}$
75	14	8	10	140		$\frac{6}{14}$	$\frac{6}{14}$	$\frac{8}{14}$	$\frac{14}{14}$

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Momentum of A before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
76	15	8	10	150		7/15	7/15	8/15	15/15
77	16	8	10	160		8/16	8/16	8/16	16/16
78	10	9	10	100		1/10	1/10	9/10	10/10
79	11	9	10	110		2/11	2/11	9/11	11/11
80	12	9	10	120		3/12	3/12	9/12	12/12
81	13	9	10	130		4/13	4/13	9/13	13/13
82	14	9	10	140		5/14	5/14	9/14	14/14
83	15	9	10	150		6/15	6/15	9/15	15/15
84	16	9	10	160		7/16	7/16	9/16	16/16
85	11	10	10	110		1/11	1/11	10/11	11/11
86	12	10	10	120		2/12	2/12	10/12	12/12
87	13	10	10	130		3/13	3/13	10/13	13/13
88	14	10	10	140		4/14	4/14	10/14	14/14
89	15	10	10	150		5/15	5/15	10/15	15/15
90	16	10	10	160		6/16	6/16	10/16	16/16
91	12	11	10	120		1/12	1/12	11/12	12/12
92	13	11	10	130		2/13	2/13	11/13	13/13
93	14	11	10	140		3/14	3/14	11/14	14/14
94	15	11	10	150		4/15	4/15	11/15	15/15
95	16	11	10	160		5/16	5/16	11/16	16/16
96	13	12	10	130		1/13	1/13	12/13	13/13
97	14	12	10	140		2/14	2/14	12/14	14/14
98	15	12	10	150		3/15	3/15	12/15	15/15
99	16	12	10	160		4/16	4/16	12/16	16/16
100	17	12	10	170		5/17	5/17	12/17	17/17

SECTION III.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE THIRD CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO THE WEIGHT OF B, AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING FORWARD IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, BUT, OF COURSE, WITH A VELOCITY LESS THAN THAT OF A.

IN every case belonging to this class, the results of collision will be determined by the following laws:—

LAW 1. A will continue to move in the same line that it moved in before collision. Both the velocity and momentum of A,—as compared with the velocity and momentum of A before collision,—will be proportional to the ratio of the velocity of B to the velocity of A before collision.

LAW 2. The momentum of B will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective velocities of A and B before collision. The other part will be equal to the momentum of B before collision.

LAW 3. The velocity of B will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective velocities of A and B before collision. The other part will be equal to the velocity of A before collision.

These laws will give the results in terms of the velocity of A before collision; and the results may all be expressed by three fractional numbers; namely, that which expresses the ratio of the velocity of B to the velocity of A before collision; that which expresses the difference between the respective velocities of A and B before collision; and that which expresses the sum of the ratio and difference of those velocities.

In case No. 1, of the following table, for example, the velocity of A, before collision, is 36 feet per second, and the weight of A is 10 pounds. Hence the momentum of A, before collision, is put down in the table as 360. The velocity of B, before collision, is 35 feet per second, and the weight of B is 10 pounds. Hence the momentum of B, before collision, is put down in the table as 350. And hence the joint momentum of A and B is 710; which is the amount that is to be accounted for, after collision, in case No. 1.

Now, according to Law 1, both the velocity and momentum of A,—as compared with what they were before collision,—will be proportional to the ratio of the velocity of B to the velocity of A before collision. In case No. 1, that ratio is $\frac{35}{36}$. Hence, both the velocity and momentum of A, after collision, are put down in the table as $\frac{35}{36}$; and the meaning of it is, that the velocity of A will be equal to thirty-five thirty-sixths of what it was before collision: that is, equal to thirty-five thirty-sixths of 36 feet per second; and that the momentum of A will be equal to thirty-five thirty-sixths of what it was before collision: that is, equal to thirty-five thirty-sixths of 360.

According to Law 2, the momentum of B will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the

difference between the respective velocities of A and B before collision; and the other part will be equal to the momentum of B before collision. In case No. 1, the difference between the respective velocities of A and B is $\frac{1}{36}$, and the momentum of B, before collision, is equal to $\frac{36}{36}$. Hence the momentum of B, after collision, is put down in the table as $\frac{1}{36}$ A + $\frac{36}{36}$ B; and the meaning of it is, that one part will be equal to one thirty-sixth of the momentum which A had before collision: that is, equal to one thirty-sixth of 360; and that the other part will be equal to thirty-six thirty-sixths of the momentum which B had before collision: that is, equal to thirty-six thirty-sixths of 350. Hence, the whole momentum of B will be equal to 360; and as the momentum of A, in this case, was equal to 350, the joint momentum of A and B is equal to 710: which is equal to their joint momentum before collision.

According to Law 3, the velocity of B will consist of two parts. One part,—as compared with the velocity of A, before collision,—will be proportional to the difference between the respective velocities of A and B before collision; and the other part will be equal to the velocity of B before collision. In case No. 1, the difference between the respective velocities of A and B before collision is $\frac{1}{36}$; and the velocity of B, before collision, is equal to $\frac{36}{36}$. Hence the velocity of B, after collision, is put down in the table as $\frac{1}{36}$ A + $\frac{36}{36}$ B; and the meaning of it is, that one part will be equal to one thirty-sixth of the velocity which A had before collision: that is, equal to one thirty-sixth of 36 feet per second; and that the other part will be equal to thirty-six thirty-sixths of the velocity which B had before collision: that is, equal to thirty-six thirty-sixths of 35 feet per second.

Now, whatever may be the respective weights and velocities of A and B, before collision, the laws above enunciated will hold good in every case in which the weight of A is equal to the weight of B, and in which B, at the time of collision, is moving forward in the same right line as that in which A is moving, with a velocity less than that of A.

TABLE I.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Velocity of B before collision in feet per second.	Momentum of A before collision.	Momentum of B before collision.	Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	10	10	36	35	360	350	$\frac{35}{36}$	$\frac{35}{36}$	$\frac{1}{36}A + \frac{36}{36}B$	$\frac{1}{36}A + \frac{36}{36}B$
2	10	10	35	34	350	340	$\frac{34}{35}$	$\frac{34}{35}$	$\frac{1}{35}A + \frac{35}{35}B$	$\frac{1}{35}A + \frac{35}{35}B$
3	10	10	34	33	340	330	$\frac{33}{34}$	$\frac{33}{34}$	$\frac{1}{34}A + \frac{34}{34}B$	$\frac{1}{34}A + \frac{34}{34}B$
4	10	10	33	32	330	320	$\frac{32}{33}$	$\frac{32}{33}$	$\frac{1}{33}A + \frac{33}{33}B$	$\frac{1}{33}A + \frac{33}{33}B$
5	10	10	32	31	320	310	$\frac{31}{32}$	$\frac{31}{32}$	$\frac{1}{32}A + \frac{32}{32}B$	$\frac{1}{32}A + \frac{32}{32}B$
6	10	10	31	30	310	300	$\frac{30}{31}$	$\frac{30}{31}$	$\frac{1}{31}A + \frac{31}{31}B$	$\frac{1}{31}A + \frac{31}{31}B$
7	10	10	30	29	300	290	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{30}A + \frac{30}{30}B$	$\frac{1}{30}A + \frac{30}{30}B$
8	10	10	29	28	290	280	$\frac{28}{29}$	$\frac{28}{29}$	$\frac{1}{29}A + \frac{29}{29}B$	$\frac{1}{29}A + \frac{29}{29}B$
9	10	10	28	27	280	270	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{1}{28}A + \frac{28}{28}B$	$\frac{1}{28}A + \frac{28}{28}B$
10	10	10	27	26	270	260	$\frac{26}{27}$	$\frac{26}{27}$	$\frac{1}{27}A + \frac{27}{27}B$	$\frac{1}{27}A + \frac{27}{27}B$
11	10	10	26	25	260	250	$\frac{25}{26}$	$\frac{25}{26}$	$\frac{1}{26}A + \frac{26}{26}B$	$\frac{1}{26}A + \frac{26}{26}B$
12	10	10	25	24	250	240	$\frac{24}{25}$	$\frac{24}{25}$	$\frac{1}{25}A + \frac{25}{25}B$	$\frac{1}{25}A + \frac{25}{25}B$
13	10	10	24	23	240	230	$\frac{23}{24}$	$\frac{23}{24}$	$\frac{1}{24}A + \frac{24}{24}B$	$\frac{1}{24}A + \frac{24}{24}B$
14	10	10	23	22	230	220	$\frac{22}{23}$	$\frac{22}{23}$	$\frac{1}{23}A + \frac{23}{23}B$	$\frac{1}{23}A + \frac{23}{23}B$
15	10	10	22	21	220	210	$\frac{21}{22}$	$\frac{21}{22}$	$\frac{1}{22}A + \frac{22}{22}B$	$\frac{1}{22}A + \frac{22}{22}B$
16	10	10	21	20	210	200	$\frac{20}{21}$	$\frac{20}{21}$	$\frac{1}{21}A + \frac{21}{21}B$	$\frac{1}{21}A + \frac{21}{21}B$
17	10	10	20	19	200	190	$\frac{19}{20}$	$\frac{19}{20}$	$\frac{1}{20}A + \frac{20}{20}B$	$\frac{1}{20}A + \frac{20}{20}B$
18	10	10	19	18	190	180	$\frac{18}{19}$	$\frac{18}{19}$	$\frac{1}{19}A + \frac{19}{19}B$	$\frac{1}{19}A + \frac{19}{19}B$
19	10	10	18	17	180	170	$\frac{17}{18}$	$\frac{17}{18}$	$\frac{1}{18}A + \frac{18}{18}B$	$\frac{1}{18}A + \frac{18}{18}B$
20	10	10	17	16	170	160	$\frac{16}{17}$	$\frac{16}{17}$	$\frac{1}{17}A + \frac{17}{17}B$	$\frac{1}{17}A + \frac{17}{17}B$
21	10	10	16	15	160	150	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{1}{16}A + \frac{16}{16}B$	$\frac{1}{16}A + \frac{16}{16}B$
22	10	10	15	14	150	140	$\frac{14}{15}$	$\frac{14}{15}$	$\frac{1}{15}A + \frac{15}{15}B$	$\frac{1}{15}A + \frac{15}{15}B$
23	10	10	14	13	140	130	$\frac{13}{14}$	$\frac{13}{14}$	$\frac{1}{14}A + \frac{14}{14}B$	$\frac{1}{14}A + \frac{14}{14}B$

TABLE II.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision.	Velocity of B before collision.	Momentum of A before collision.	Momentum of B before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	10	10	36	1	360	10		$\frac{1}{36}$	$\frac{1}{36}$	$\frac{35}{36}A+\frac{36}{36}B$	$\frac{35}{36}A+\frac{36}{36}B$
2	10	10	36	2	360	20		$\frac{2}{36}$	$\frac{2}{36}$	$\frac{34}{36}A+\frac{36}{36}B$	$\frac{34}{16}A+\frac{36}{36}B$
3	10	10	36	3	360	30		$\frac{3}{36}$	$\frac{3}{36}$	$\frac{33}{36}A+\frac{36}{36}B$	$\frac{33}{36}A+\frac{36}{36}B$
4	10	10	36	4	360	40		$\frac{4}{36}$	$\frac{4}{36}$	$\frac{32}{36}A+\frac{36}{36}B$	$\frac{32}{36}A+\frac{36}{36}B$
5	10	10	36	5	360	50		$\frac{5}{36}$	$\frac{5}{36}$	$\frac{31}{36}A+\frac{36}{36}B$	$\frac{31}{36}A+\frac{36}{36}B$
6	10	10	36	6	360	60		$\frac{6}{36}$	$\frac{6}{36}$	$\frac{30}{36}A+\frac{36}{36}B$	$\frac{10}{16}A+\frac{36}{36}B$
7	10	10	36	7	360	70		$\frac{7}{36}$	$\frac{7}{36}$	$\frac{29}{36}A+\frac{36}{36}B$	$\frac{29}{36}A+\frac{36}{36}B$
8	10	10	36	8	360	80		$\frac{8}{36}$	$\frac{8}{36}$	$\frac{28}{36}A+\frac{36}{36}B$	$\frac{28}{36}A+\frac{36}{36}B$
9	10	10	36	9	360	90		$\frac{9}{36}$	$\frac{9}{36}$	$\frac{27}{36}A+\frac{36}{36}B$	$\frac{27}{36}A+\frac{36}{36}B$
10	10	10	36	10	360	100		$\frac{10}{36}$	$\frac{10}{36}$	$\frac{26}{36}A+\frac{36}{36}B$	$\frac{6}{6}A+\frac{36}{36}B$
11	10	10	36	11	360	110		$\frac{11}{36}$	$\frac{11}{36}$	$\frac{25}{36}A+\frac{36}{36}B$	$\frac{25}{36}A+\frac{36}{36}B$
12	10	10	36	12	360	120		$\frac{12}{36}$	$\frac{12}{36}$	$\frac{24}{36}A+\frac{36}{36}B$	$\frac{24}{36}A+\frac{36}{36}B$
13	10	10	36	13	360	130		$\frac{13}{36}$	$\frac{13}{36}$	$\frac{23}{36}A+\frac{36}{36}B$	$\frac{23}{36}A+\frac{36}{36}B$
14	10	10	36	14	360	140		$\frac{14}{36}$	$\frac{14}{36}$	$\frac{22}{36}A+\frac{36}{36}B$	$\frac{22}{36}A+\frac{36}{36}B$
15	10	10	36	15	360	150		$\frac{15}{36}$	$\frac{15}{36}$	$\frac{21}{36}A+\frac{36}{36}B$	$\frac{21}{36}A+\frac{36}{36}B$
16	10	10	36	16	360	160		$\frac{16}{36}$	$\frac{16}{36}$	$\frac{20}{36}A+\frac{36}{36}B$	$\frac{20}{36}A+\frac{36}{36}B$
17	10	10	36	17	360	170		$\frac{17}{36}$	$\frac{17}{36}$	$\frac{19}{36}A+\frac{36}{36}B$	$\frac{19}{36}A+\frac{36}{36}B$
18	10	10	36	18	360	180		$\frac{18}{36}$	$\frac{18}{36}$	$\frac{18}{36}A+\frac{36}{36}B$	$\frac{18}{36}A+\frac{36}{36}B$
19	10	10	36	19	360	190		$\frac{19}{36}$	$\frac{19}{36}$	$\frac{17}{36}A+\frac{36}{36}B$	$\frac{17}{36}A+\frac{36}{36}B$
20	10	10	36	20	360	200		$\frac{20}{36}$	$\frac{20}{36}$	$\frac{16}{36}A+\frac{36}{36}B$	$\frac{16}{36}A+\frac{36}{36}B$
21	10	10	36	21	360	210		$\frac{21}{36}$	$\frac{21}{36}$	$\frac{15}{36}A+\frac{36}{36}B$	$\frac{15}{36}A+\frac{36}{36}B$
22	10	10	36	22	360	220		$\frac{22}{36}$	$\frac{22}{36}$	$\frac{14}{36}A+\frac{36}{36}B$	$\frac{14}{36}A+\frac{36}{36}B$
23	10	10	36	23	360	230		$\frac{23}{36}$	$\frac{23}{36}$	$\frac{13}{36}A+\frac{36}{36}B$	$\frac{13}{36}A+\frac{36}{36}B$

SECTION IV.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE FOURTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS LESS THAN THE WEIGHT OF B, AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING FORWARD IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, WITH A VELOCITY LESS THAN THAT OF A.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A, after collision, will have a velocity in two directions; namely, one in the direction in which it moved before collision, and one in direct opposition to that. The velocity of A in its given direction,—as compared with its own velocity before collision,—will be proportional to the ratio of the velocity of B before collision to the velocity of A before collision. The velocity of A in opposition to its given direction,—as compared with its own velocity before collision,—will be proportional to the difference between the respective weights of A and B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision.

Law 2. The momentum of A, in the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the ratio of the velocity of B before collision to the velocity of A before collision. The momentum of A in opposition to the given direction

of A,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision.

Law 3. B will continue to move in its own given direction. The momentum of B will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision. The other part will be equal to the momentum of B before collision.

Law 4. The velocity of B will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B, minus a quantity proportional to the square of the ratio of the velocity of B before collision to the velocity of A before collision. The other part will be equal to the velocity of B before collision.

These Laws will give the results of collision in terms of the weight of B, and the velocity of A before collision. And the results may all be expressed by four fractional numbers; namely, that which expresses the ratio of the weight of A to the weight of B; that which expresses the difference between the respective weights of A and B; that which expresses the sum of the ratio and difference of the weights of A and B; and that which expresses the ratio of the velocity of B before collision to the velocity of A before collision.

In case No. 5, of the following Table, for example, the weight of A is 2 pounds, and the velocity of A before collision ten feet per second. Hence, the momentum of

A, before collision, is put down in the Table as 20. The weight of B is 6 pounds, and the velocity of B is 2 feet per second. Hence, the momentum of B, before collision, is put down in the Table as 12; and the joint momentum of A and B as 32: which is the amount of the momentum that is to be accounted for after collision.

Now, according to Law 1, the velocity of A in its own given direction,—as compared with its own given velocity before collision,—will be proportional to the ratio of the velocity of B before collision to the velocity of A before collision. In case No 5, that ratio is $\frac{2}{10}$. Hence, the velocity of A in its own given direction is put down in the Table as $\frac{2}{10}$ A; and the meaning of it is, that it will be equal to two tenths of the velocity which A had before collision: that is, equal to two tenths of ten feet per second. According to the same Law, the velocity of A, in opposition to its own given direction,—as compared with its own velocity before collision,—will be proportional to the difference between the respective weights of A and B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the ratio of the velocities $\frac{2}{10}$. Hence, the velocity of A after collision, in opposition to its own given direction, is put down in the table as $\frac{4}{6}A-\frac{2}{10}$; and the meaning of it is, that it will be equal to four sixths of the velocity which A had before collision, minus two tenths of four sixths: that is, equal to four sixths of 10 feet per second: equal to $5\frac{1}{3}$ feet. Hence, the whole velocity of A in both directions will be equal to $7\frac{1}{3}$ feet per second.

According to Law 2, the momentum of A in the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the ratio of the

velocity of B before collision to the velocity of A before collision. In case No. 5, that ratio is $\frac{2}{10}$. Hence, the momentum of A, in opposition to the given direction of A, is put down in the table as $\frac{2}{10}$ A; and the meaning of it is, that it will be equal to two tenths of the momentum which A had before collision: that is, equal to two-tenths of 20; equal to 4. According to the same Law, the momentum of A, in opposition to the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the ratio of the velocities $\frac{2}{10}$. Hence, the momentum of A, in opposition to the given direction of A, is put down in the table as $\frac{4}{6}$ A $-$ $\frac{2}{10}$; and the meaning of it is, that it will be equal to four sixths of the momentum which A had before collision, minus two tenths of four sixths: that is equal to four sixths of 20: equal to $10\frac{2}{3}$. And, as A had a momentum equal to 4, in the other direction, the whole momentum of A after collision will be equal to $14\frac{2}{3}$.

According to Law 3, one part of the momentum of B, after collision,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision. The other part will be equal to the momentum which B had before collision. In case No. 5, the ratio of the weights is $\frac{2}{6}$; the ratio of the velocities $\frac{2}{10}$; and the momentum of B before collision $\frac{5}{6}$. Hence, the momentum of B, after collision, is put down in the table as $\frac{2}{6}$ A $-$ $\frac{2}{10}$ $+$ $\frac{5}{6}$ B. The meaning

of the $\frac{2}{6}$ A $-\frac{2}{10}$, is, that one portion of the momentum of B will be equal to two sixths of the momentum which A had before collision, minus two tenths of two sixths: that is, equal to two sixths of 20, minus two tenths of two sixths of 20: equal to $5\frac{1}{3}$. The meaning of the $\frac{6}{6}$ B is, that the other portion will be equal to six sixths of the momentum which B had before collision; that is, equal to six sixths of 12. Hence, the whole momentum of B after collision will be equal to $17\frac{1}{3}$; which, added to the momentum of A, $14\frac{2}{3}$, will make 32: which is equal to the whole of the momentum that was to be accounted for after collision.

According to Law 4, one part of the velocity of B after collision,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B, minus a quantity proportional to the ratio of the velocity of B before collision to the velocity of A before collision. And the other part will be equal to the velocity of B before collision. In case No. 5, the ratio of the weights is $\frac{2}{6}$; the ratio of the velocities $\frac{2}{10}$; and the velocity of B $\frac{6}{6}$. Hence, the velocity of B after collision, is put down in the table as $\frac{2}{6}$ A$^2 - \frac{2}{10}$, + $\frac{6}{6}$ B. The meaning of the $\frac{2}{6}$ A$^2 - \frac{2}{10}$, is, that one part will be equal to four thirty-sixths of the velocity which A had before collision, minus two tenths of four thirty-sixths: that is, equal to four thirty-sixths of 10 feet per second, minus two tenths of four thirty-sixths of 10 feet: equal to eight ninths of one foot per second. The meaning of the $\frac{6}{6}$ B is, that the other part will be equal to six sixths of the velocity which B had before collision: that is, equal to six sixths of 2 feet per second. Hence, the whole velocity of B will be equal to $2\frac{8}{9}$ feet per second.

Now, whatever may be the weight of A, or the weight

of B; or whatever may be the velocity of A, or the velocity of B before collision; the Laws above enunciated will hold good in every case in which the weight of A is less than the weight of B; in which the velocity of B is less than the velocity of A; and in which B, at the time of collision, is moving forward in the same right line as that in which A is moving.

TABLE.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision.	Velocity of B before collision.	Momentum of A before collision.	Momentum of B before collision.	Momentum of A and B before collision.		Velocity of A after collision in the given direction of A.	Velocity of A after collision in opposition to the given direction of A.	Momentum of A after collision in the given direction of A.	Momentum of A after collision in opposition to the given direction of A.	Momentum of B after collision.	Velocity of B after collision.
1	1	2	10	1	10	2	12		$\frac{1}{10}A$	$\frac{1}{2}A-\frac{1}{10}$	$\frac{1}{10}A$	$\frac{1}{2}A-\frac{1}{10}$	$\frac{1}{2}A-\frac{1}{10}, +\frac{2}{2}B$	$\frac{1}{2}A^2-\frac{1}{10}, +\frac{2}{2}B$
2	1	3	10	1	10	3	13		$\frac{1}{10}A$	$\frac{2}{3}A-\frac{1}{10}$	$\frac{1}{10}A$	$\frac{2}{3}A-\frac{1}{10}$	$\frac{1}{3}A-\frac{1}{10}, +\frac{3}{3}B$	$\frac{1}{3}A^2-\frac{1}{10}, +\frac{3}{3}B$
3	1	4	10	2	10	8	18		$\frac{2}{10}A$	$\frac{3}{4}A-\frac{2}{10}$	$\frac{2}{10}A$	$\frac{3}{4}A-\frac{2}{10}$	$\frac{1}{4}A-\frac{2}{10}, +\frac{4}{4}B$	$\frac{1}{4}A^2-\frac{2}{10}, +\frac{4}{4}B$
4	1	5	10	2	10	10	20		$\frac{2}{10}A$	$\frac{4}{5}A-\frac{2}{10}$	$\frac{2}{10}A$	$\frac{4}{5}A-\frac{2}{10}$	$\frac{1}{5}A-\frac{2}{10}, +\frac{5}{5}B$	$\frac{1}{5}A^2-\frac{2}{10}, +\frac{5}{5}B$
5	2	6	10	2	20	12	32		$\frac{2}{10}A$	$\frac{4}{6}A-\frac{2}{10}$	$\frac{2}{10}A$	$\frac{4}{6}A-\frac{2}{10}$	$\frac{2}{6}A-\frac{2}{10}, +\frac{6}{6}B$	$\frac{2}{6}A^2-\frac{2}{10}, +\frac{6}{6}B$
6	2	7	10	3	20	21	41		$\frac{3}{10}A$	$\frac{5}{7}A-\frac{3}{10}$	$\frac{3}{10}A$	$\frac{5}{7}A-\frac{3}{10}$	$\frac{2}{7}A-\frac{3}{10}, +\frac{7}{7}B$	$\frac{2}{7}A^2-\frac{3}{10}, +\frac{7}{7}B$
7	2	8	10	3	20	24	44		$\frac{3}{10}A$	$\frac{6}{8}A-\frac{3}{10}$	$\frac{3}{10}A$	$\frac{6}{8}A-\frac{3}{10}$	$\frac{2}{8}A-\frac{3}{10}, +\frac{8}{8}B$	$\frac{2}{8}A^2-\frac{3}{10}, +\frac{8}{8}B$
8	2	9	10	3	20	27	47		$\frac{3}{10}A$	$\frac{7}{9}A-\frac{3}{10}$	$\frac{3}{10}A$	$\frac{7}{9}A-\frac{3}{10}$	$\frac{2}{9}A-\frac{3}{10}, +\frac{9}{9}B$	$\frac{2}{9}A^2-\frac{3}{10}, +\frac{9}{9}B$
9	2	10	10	4	20	40	60		$\frac{4}{10}A$	$\frac{8}{10}A-\frac{4}{10}$	$\frac{4}{10}A$	$\frac{8}{10}A-\frac{4}{10}$	$\frac{2}{10}A-\frac{4}{10}, +\frac{10}{10}B$	$\frac{2}{10}A^2-\frac{4}{10}, +\frac{10}{10}B$
10	3	11	10	4	30	44	74		$\frac{4}{10}A$	$\frac{8}{11}A-\frac{4}{10}$	$\frac{4}{10}A$	$\frac{8}{11}A-\frac{4}{10}$	$\frac{3}{11}A-\frac{4}{10}, +\frac{11}{11}B$	$\frac{3}{11}A^2-\frac{4}{10}, +\frac{11}{11}B$
11	3	12	10	4	30	48	78		$\frac{4}{10}A$	$\frac{9}{12}A-\frac{4}{10}$	$\frac{4}{10}A$	$\frac{9}{12}A-\frac{4}{10}$	$\frac{3}{12}A-\frac{4}{10}, +\frac{12}{12}B$	$\frac{3}{12}A^2-\frac{4}{10}, +\frac{12}{12}B$
12	3	13	10	4	30	52	82		$\frac{4}{10}A$	$\frac{10}{13}A-\frac{4}{10}$	$\frac{4}{10}A$	$\frac{10}{13}A-\frac{4}{10}$	$\frac{3}{13}A-\frac{4}{10}, +\frac{13}{13}B$	$\frac{3}{13}A^2-\frac{4}{10}, +\frac{13}{13}B$

13	3	14	10	5	30	70	100	$\frac{5}{10}A$	$\frac{11}{14}A-\frac{5}{10}$	$\frac{5}{10}A$	$\frac{11}{14}A-\frac{5}{10}$	$\frac{3}{14}A-\frac{5}{10}, +\frac{14}{14}B$	$\frac{3}{14}A^2-\frac{5}{10}, +\frac{14}{14}B$
14	3	15	10	5	30	75	105	$\frac{5}{10}A$	$\frac{12}{15}A-\frac{5}{10}$	$\frac{5}{10}A$	$\frac{12}{15}A-\frac{5}{10}$	$\frac{3}{15}A-\frac{5}{10}, +\frac{15}{15}B$	$\frac{3}{15}A^2-\frac{5}{10}, +\frac{15}{15}B$
15	4	16	10	5	40	80	120	$\frac{5}{10}A$	$\frac{12}{16}A-\frac{5}{10}$	$\frac{5}{10}A$	$\frac{12}{16}A-\frac{5}{10}$	$\frac{4}{16}A-\frac{5}{10}, +\frac{16}{16}B$	$\frac{4}{16}A^2-\frac{5}{10}, +\frac{16}{16}B$
16	4	17	10	5	40	85	125	$\frac{5}{10}A$	$\frac{13}{17}A-\frac{5}{10}$	$\frac{5}{10}A$	$\frac{13}{17}A-\frac{5}{10}$	$\frac{4}{17}A-\frac{5}{10}, +\frac{17}{17}B$	$\frac{4}{17}A^2-\frac{5}{10}, +\frac{17}{17}B$
17	4	18	10	5	40	90	130	$\frac{5}{10}A$	$\frac{14}{18}A-\frac{5}{10}$	$\frac{5}{10}A$	$\frac{14}{18}A-\frac{5}{10}$	$\frac{4}{18}A-\frac{5}{10}, +\frac{18}{18}B$	$\frac{4}{18}A^2-\frac{5}{10}, +\frac{18}{18}B$
18	4	19	10	6	40	114	154	$\frac{6}{10}A$	$\frac{15}{19}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{15}{19}A-\frac{6}{10}$	$\frac{4}{19}A-\frac{6}{10}, +\frac{19}{19}B$	$\frac{4}{19}A^2-\frac{6}{10}, +\frac{19}{19}B$
19	4	20	10	6	40	120	160	$\frac{6}{10}A$	$\frac{16}{20}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{16}{20}A-\frac{6}{10}$	$\frac{4}{20}A-\frac{6}{10}, +\frac{20}{20}B$	$\frac{4}{20}A^2-\frac{6}{10}, +\frac{20}{20}B$
20	5	21	10	6	50	126	176	$\frac{6}{10}A$	$\frac{16}{21}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{16}{21}A-\frac{6}{10}$	$\frac{5}{21}A-\frac{6}{10}, +\frac{21}{21}B$	$\frac{5}{21}A^2-\frac{6}{10}, +\frac{21}{21}B$
21	5	22	10	6	50	132	182	$\frac{6}{10}A$	$\frac{17}{22}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{17}{22}A-\frac{6}{10}$	$\frac{5}{22}A-\frac{6}{10}, +\frac{22}{22}B$	$\frac{5}{22}A^2-\frac{6}{10}, +\frac{22}{22}B$
22	5	23	10	6	50	138	188	$\frac{6}{10}A$	$\frac{18}{23}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{18}{23}A-\frac{6}{10}$	$\frac{5}{23}A-\frac{6}{10}, +\frac{23}{23}B$	$\frac{5}{23}A^2-\frac{6}{10}, +\frac{23}{23}B$
23	5	24	10	6	50	144	194	$\frac{6}{10}A$	$\frac{19}{24}A-\frac{6}{10}$	$\frac{6}{10}A$	$\frac{19}{24}A-\frac{6}{10}$	$\frac{5}{24}A-\frac{6}{10}, +\frac{24}{24}B$	$\frac{5}{24}A^2-\frac{6}{10}, +\frac{24}{24}B$
24	5	25	10	7	50	175	225	$\frac{7}{10}A$	$\frac{20}{25}A-\frac{7}{10}$	$\frac{7}{10}A$	$\frac{20}{25}A-\frac{7}{10}$	$\frac{5}{25}A-\frac{7}{10}, +\frac{25}{25}B$	$\frac{5}{25}A^2-\frac{7}{10}, +\frac{25}{25}B$
25	6	26	10	7	60	182	242	$\frac{7}{10}A$	$\frac{20}{26}A-\frac{7}{10}$	$\frac{7}{10}A$	$\frac{20}{26}A-\frac{7}{10}$	$\frac{6}{26}A-\frac{7}{10}, +\frac{26}{26}B$	$\frac{6}{26}A^2-\frac{7}{10}, +\frac{26}{26}B$
26	6	27	10	7	60	189	249	$\frac{7}{10}A$	$\frac{21}{27}A-\frac{7}{10}$	$\frac{7}{10}A$	$\frac{21}{27}A-\frac{7}{10}$	$\frac{6}{27}A-\frac{7}{10}, +\frac{27}{27}B$	$\frac{6}{27}A^2-\frac{7}{10}, +\frac{27}{27}B$
27	6	28	10	7	60	196	256	$\frac{7}{10}A$	$\frac{22}{28}A-\frac{7}{10}$	$\frac{7}{10}A$	$\frac{22}{28}A-\frac{7}{10}$	$\frac{6}{28}A-\frac{7}{10}, +\frac{28}{28}B$	$\frac{6}{28}A^2-\frac{7}{10}, +\frac{28}{28}B$
28	6	29	10	8	60	232	292	$\frac{8}{10}A$	$\frac{23}{29}A-\frac{8}{10}$	$\frac{8}{10}A$	$\frac{23}{29}A-\frac{8}{10}$	$\frac{6}{29}A-\frac{8}{10}, +\frac{29}{29}B$	$\frac{6}{29}A^2-\frac{8}{10}, +\frac{29}{29}B$
29	6	30	10	8	60	240	300	$\frac{8}{10}A$	$\frac{24}{30}A-\frac{8}{10}$	$\frac{8}{10}A$	$\frac{24}{30}A-\frac{8}{10}$	$\frac{6}{30}A-\frac{8}{10}, +\frac{30}{30}B$	$\frac{6}{30}A^2-\frac{8}{10}, +\frac{30}{30}B$
30	7	31	10	8	70	248	318	$\frac{8}{10}A$	$\frac{24}{31}A-\frac{8}{10}$	$\frac{8}{10}A$	$\frac{24}{31}A-\frac{8}{10}$	$\frac{7}{31}A-\frac{8}{10}, +\frac{31}{31}B$	$\frac{7}{31}A^2-\frac{8}{10}, +\frac{31}{31}B$

SECTION V.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE FIFTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS GREATER THAN THE WEIGHT OF B, AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING FORWARD IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, WITH A VELOCITY LESS THAN THAT OF A.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will move in the same direction after as before collision. The velocity of A will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and B. The other part,—as compared with the velocity of B before collision,—will be proportional to the ratio of the weight of B to the weight of A.

Law 2. The momentum of A will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B. The other part will be equal to the momentum of B before collision.

Law 3. The momentum of B, after collision,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of B to the weight of A.

Law 4. The velocity of B, after collision, will be equal to the velocity of A before collision.

These Laws will give the results of collision in terms of the weight of A; and the results, in each and every case, may all be expressed by three fractional numbers; namely, that which expresses the ratio of the weight of B to the weight of A; that which expresses the difference between the respective weights of A and B; and that which expresses the sum of the ratio and difference of the respective weights of A and B.

In case No. 6, of the following Table, for example, the weight of A is 10 pounds, and the given velocity of A is 10 feet per second. Hence, the momentum of A, before collision, is put down in the Table as 100. The weight of B is 6 pounds; and the given velocity of B is 6 feet per second. Hence, the momentum of B, before collision, is put down in the Table as 36; and the joint momentum of A and B as 136: which is the amount of the momentum that is to be accounted for after collision, in case No. 6.

Now, according to Law 1, one part of the velocity of A after collision,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other part,—as compared with the velocity of B before collision,—will be proportional to the ratio of the weight of B to the weight of A. In case No. 6, the difference between the weights is $\frac{4}{10}$, and the ratio of the weight of B to the weight of A is $\frac{6}{10}$. Hence, the velocity of A, after collision, is put down in the Table as $\frac{4}{10}A + \frac{6}{10}B$. The meaning of the $\frac{4}{10}A$, is, that one part will be equal to four tenths of the velocity which A had before collision: that is, equal to four tenths of 10 feet per second. The meaning of the $\frac{6}{10}B$, is, that the other part will be equal to six tenths of the velocity which B had before collision: that is, equal to six tenths of six feet per second. Now,

four tenths of 10 feet are equal to 4 feet; and six tenths of 6 feet are equal to $3\frac{6}{10}$ feet. Hence, the whole velocity of A after collision, in case No. 6, will be equal to $7\frac{6}{10}$ feet per second.

According to Law 2, one part of the momentum of A, after collision,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other part equal to the momentum of B before collision. In case No. 6, the difference between the weights is $\frac{4}{10}$, and the momentum of B is $\frac{10}{10}$. Hence, the momentum of A, after collision, is put down in the Table as $\frac{4}{10}A + \frac{10}{10}B$. The meaning of the $\frac{4}{10}A$, is, that one part will be equal to four tenths of the momentum which A had before collision: that is, equal to four tenths of 100. The meaning of the $\frac{10}{10}B$, is, that the other part will be equal to ten tenths of the momentum which B had before collision: that is, equal to ten tenths of 36. Now, four tenths of 100 are equal to 40, and ten tenths of 36 are equal to 36. Hence, the whole momentum of A, after collision, will be equal to 76.

According to Law 3, the momentum of B after collision,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of B to the weight of A. In case No. 6, that ratio is $\frac{6}{10}$. Hence, the momentum of B after collision, is put down in the Table as $\frac{6}{10}A$; and the meaning of it is, that it will be equal to six tenths of the momentum which A had before collision: that is, equal to six tenths of 100: equal to 60.

According to Law 4, the velocity of B after collision will be equal to the velocity of A before collision. Hence, the velocity of B is put down in the Table as $\frac{10}{10}A$, and the meaning of it is, that it will be equal to ten tenths of

the velocity which A had before collision: that is, equal to ten tenths of ten feet per second.

Now, whatever may be the weight of A, or the weight of B; or whatever may be the velocity of A, or the velocity of B, before collision; the Laws above enunciated will hold good in every case in which the weight of A is greater than, or equal to, the weight of B; and in which B, at the time of collision, is moving forward in the same right line as that in which A is moving, with a velocity less than that of A.

TABLE.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Velocity of B before collision in feet per second.	Momentum of A before collision.	Momentum of B before collision.	Momentum of A and B before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	10	1	10	1	100	1	101		$\frac{9}{10}A+\frac{1}{10}B$	$\frac{9}{10}A+\frac{10}{10}B$	$\frac{1}{10}A$	$\frac{10}{10}A$
2	10	2	10	2	100	4	104		$\frac{8}{10}A+\frac{2}{10}B$	$\frac{8}{10}A+\frac{10}{10}B$	$\frac{2}{10}A$	$\frac{10}{10}A$
3	10	3	10	3	100	9	109		$\frac{7}{10}A+\frac{3}{10}B$	$\frac{7}{10}A+\frac{10}{10}B$	$\frac{3}{10}A$	$\frac{10}{10}A$
4	10	4	10	4	100	16	116		$\frac{6}{10}A+\frac{4}{10}B$	$\frac{6}{10}A+\frac{10}{10}B$	$\frac{4}{10}A$	$\frac{10}{10}A$
5	10	5	10	5	100	25	125		$\frac{5}{10}A+\frac{5}{10}B$	$\frac{5}{10}A+\frac{10}{10}B$	$\frac{5}{10}A$	$\frac{10}{10}A$
6	10	6	10	6	100	36	136		$\frac{4}{10}A+\frac{6}{10}B$	$\frac{4}{10}A+\frac{10}{10}B$	$\frac{6}{10}A$	$\frac{10}{10}A$
7	10	7	10	7	100	49	149		$\frac{3}{10}A+\frac{7}{10}B$	$\frac{3}{10}A+\frac{10}{10}B$	$\frac{7}{10}A$	$\frac{10}{10}A$
8	10	8	10	8	100	64	164		$\frac{2}{10}A+\frac{8}{10}B$	$\frac{2}{10}A+\frac{10}{10}B$	$\frac{8}{10}A$	$\frac{10}{10}A$
9	10	9	10	9	100	81	. 181		$\frac{1}{10}A+\frac{9}{10}B$	$\frac{1}{10}A+\frac{10}{10}B$	$\frac{9}{10}A$	$\frac{10}{10}A$
10	10	10	10	8	100	80	180		$\frac{0}{10}A+\frac{10}{10}B$	$\frac{0}{10}A+\frac{10}{10}B$	$\frac{10}{10}A$	$\frac{10}{10}A$
11	10	9	10	7	100	63	163		$\frac{1}{10}A+\frac{9}{10}B$	$\frac{1}{10}A+\frac{10}{10}B$	$\frac{9}{10}A$	$\frac{10}{10}A$
12	10	8	10	6	100	48	148		$\frac{2}{10}A+\frac{8}{10}B$	$\frac{2}{10}A+\frac{10}{10}B$	$\frac{8}{10}A$	$\frac{10}{10}A$

13	10	7	10	5	100	35	135		$\frac{3}{10}A+\frac{7}{10}B$	$\frac{3}{10}A+\frac{10}{10}B$	$\frac{7}{10}A$	$\frac{10}{10}A$
14	10	6	10	4	100	24	124		$\frac{4}{10}A+\frac{6}{10}B$	$\frac{4}{10}A+\frac{10}{10}B$	$\frac{6}{10}A$	$\frac{10}{10}A$
15	10	5	10	3	100	15	115		$\frac{5}{10}A+\frac{5}{10}B$	$\frac{5}{10}A+\frac{10}{10}B$	$\frac{5}{10}A$	$\frac{10}{10}A$
16	10	4	10	2	100	8	108		$\frac{9}{10}A+\frac{4}{10}B$	$\frac{6}{10}A+\frac{10}{10}B$	$\frac{4}{10}A$	$\frac{10}{10}A$
17	10	3	10	1	100	3	103		$\frac{7}{10}A+\frac{3}{10}B$	$\frac{7}{10}A+\frac{10}{10}B$	$\frac{3}{10}A$	$\frac{10}{10}A$
18	10	2	10	2	100	4	104		$\frac{8}{10}A+\frac{2}{10}B$	$\frac{8}{10}A+\frac{10}{10}B$	$\frac{2}{10}A$	$\frac{10}{10}A$
19	10	1	10	3	100	3	103		$\frac{9}{10}A+\frac{1}{10}B$	$\frac{9}{10}A+\frac{10}{10}B$	$\frac{1}{10}A$	$\frac{10}{10}A$
20	10	2	10	4	100	8	108		$\frac{8}{10}A+\frac{2}{10}B$	$\frac{8}{10}A+\frac{10}{10}B$	$\frac{2}{10}A$	$\frac{10}{10}A$
21	10	3	10	5	100	15	115		$\frac{7}{10}A+\frac{3}{10}B$	$\frac{7}{10}A+\frac{10}{10}B$	$\frac{3}{10}A$	$\frac{10}{10}A$
22	10	4	10	6	100	24	124		$\frac{6}{10}A+\frac{4}{10}B$	$\frac{6}{10}A+\frac{10}{10}B$	$\frac{4}{10}A$	$\frac{10}{10}A$
23	10	5	10	7	100	35	135		$\frac{5}{10}A+\frac{5}{10}B$	$\frac{5}{10}A+\frac{10}{10}B$	$\frac{5}{10}A$	$\frac{10}{10}A$
24	10	6	10	8	100	48	148		$\frac{4}{10}A+\frac{6}{10}B$	$\frac{4}{10}A+\frac{10}{10}B$	$\frac{6}{10}A$	$\frac{10}{10}A$
25	10	7	10	9	100	63	163		$\frac{3}{10}A+\frac{7}{10}B$	$\frac{3}{10}A+\frac{10}{10}B$	$\frac{7}{10}A$	$\frac{10}{10}A$
26	10	8	10	8	100	64	164		$\frac{2}{10}A+\frac{8}{10}B$	$\frac{2}{10}A+\frac{10}{10}B$	$\frac{8}{10}A$	$\frac{10}{10}A$
27	10	9	10	7	100	63	163		$\frac{1}{10}A+\frac{9}{10}B$	$\frac{1}{10}A+\frac{10}{10}B$	$\frac{9}{10}A$	$\frac{10}{10}A$
28	10	10	10	6	100	60	160		$\frac{0}{10}A+\frac{10}{10}B$	$\frac{0}{10}A+\frac{10}{10}B$	$\frac{10}{10}A$	$\frac{10}{10}A$
29	10	9	10	5	100	45	145		$\frac{1}{10}A+\frac{9}{10}B$	$\frac{1}{10}A+\frac{10}{10}B$	$\frac{9}{10}A$	$\frac{10}{10}A$
30	10	8	10	4	100	32	132		$\frac{2}{10}A+\frac{8}{10}B$	$\frac{2}{10}A+\frac{10}{10}B$	$\frac{8}{10}A$	$\frac{10}{10}A$

SECTION VI.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE SIXTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO THE WEIGHT OF B, AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, BUT IN AN OPPOSITE DIRECTION.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity and momentum of A, after collision, will be equal to the given velocity and momentum of B before collision.

LAW 2. B will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity and momentum of B, after collision, will be equal to the given velocity and momentum of A before collision.

The cases belonging to this class are all included in the Seventh, and also in the Eighth class; and they belong with equal propriety to each of them. They form the connecting link between those two classes, and therefore come under the Laws of both. But as they may also be regarded as forming a class by themselves, separate Laws have been assigned to them.

SECTION VII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE SEVENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR LESS THAN, THE WEIGHT OF B, AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, BUT IN AN OPPOSITE DIRECTION.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity of A will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and B. The other part will be equal to the velocity of B before collision.

LAW 2. The momentum of A, after collision, will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B. The other part,—as compared with the momentum of B before collision,—will be proportional to the ratio of the weight of A to the weight of B.

LAW 3. The momentum of B in the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B. The momentum of B, in the given

direction of B,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of A and B.

Law 4. B will have a velocity in the given direction of A, which,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B. B will also have a velocity in its own given direction, which,—as compared with the velocity of B before collision,—will be proportional to the difference between the respective weights of A and B.

These Laws will give the results of collision in terms of the weight of B; and the results may all be expressed by four fractional numbers; namely, that which expresses the ratio of the weight of A to the weight of B; that which expresses the difference between the respective weights of A and B; that which expresses the sum of the ratio and difference of the weights; and that which expresses the square of the ratio of the weight of A to the weight of B.

In case No. 5, of the following Table, for example, the weight of A is 2 pounds, and the velocity of A, before collision, 5 feet per second. Hence, the momentum of A before collision, is put down in the Table as 10. The weight of B is 6 pounds, and the velocity of B, before collision, is 10 feet per second. Hence, the momentum of B, before collision, is put down in the Table as 60; and the joint momentum of A and B as 70: which is the amount of the momentum that is to be accounted for after collision, in case No. 5.

Now, according to Law 1, one part of the velocity of A after collision,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other

part equal to the velocity of B before collision. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the velocity of B is $\frac{6}{6}$. Hence, the velocity of A after collision, is put down in the Table as $\frac{4}{6}A + \frac{6}{6}B$. The meaning of the $\frac{4}{6}$ A, is, that one part will be equal to four sixths of the velocity which A had before collision; that is, equal to four sixths of 5 feet per second. The meaning of the $\frac{6}{6}$ B is, that the other part will be equal to six sixths of the velocity which B had before collision: that is, equal to six sixths of 10 feet per second.

According to Law 2, one part of the momentum of A, after collision,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other part,—as compared with the momentum of B before collision,—will be proportional to the ratio of the weight of A to the weight of B. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the ratio of the weight of A to the weight of B is $\frac{2}{6}$. Hence, the momentum of A after collision, is put down in the table as $\frac{4}{6}A + \frac{2}{6}B$. The meaning of the $\frac{4}{6}A$, is, that one part will be equal to four sixths of the momentum which A had before collision: that is, equal to four sixths of 10. The meaning of the $\frac{2}{6}B$ is, that the other part will be equal to two sixths of the momentum which B had before collision: that is, equal to two sixths of 60.

According to Law 3, B, after collision, will have a momentum, in the given direction of A, which,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B; and a momentum in the given direction of B, which,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of A and B. In case No. 5, the

direction of B,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of A and B.

LAW 4. B will have a velocity in the given direction of A, which,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B. B will also have a velocity in its own given direction, which,—as compared with the velocity of B before collision,—will be proportional to the difference between the respective weights of A and B.

These Laws will give the results of collision in terms of the weight of B; and the results may all be expressed by four fractional numbers; namely, that which expresses the ratio of the weight of A to the weight of B; that which expresses the difference between the respective weights of A and B; that which expresses the sum of the ratio and diffefence of the weights; and that which expresses the square of the ratio of the weight of A to the weight of B.

In case No. 5, of the following Table, for example, the weight of A is 2 pounds, and the velocity of A, before collision, 5 feet per second. Hence, the momentum of A before collision, is put down in the Table as 10. The weight of B is 6 pounds, and the velocity of B, before collision, is 10 feet per second. Hence, the momentum of B, before collision, is put down in the Table as 60; and the joint momentum of A and B as 70: which is the amount of the momentum that is to be accounted for after collision, in case No. 5.

Now, according to Law 1, one part of the velocity of A after collision,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other

part equal to the velocity of B before collision. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the velocity of B is $\frac{6}{6}$. Hence, the velocity of A after collision, is put down in the Table as $\frac{4}{6}A + \frac{6}{6}B$. The meaning of the $\frac{4}{6}$ A, is, that one part will be equal to four sixths of the velocity which A had before collision: that is, equal to four sixths of 5 feet per second. The meaning of the $\frac{6}{6}$ B is, that the other part will be equal to six sixths of the velocity which B had before collision: that is, equal to six sixths of 10 feet per second.

According to Law 2, one part of the momentum of A, after collision,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of A and B; and the other part,—as compared with the momentum of B before collision,—will be proportional to the ratio of the weight of A to the weight of B. In case No. 5, the difference between the weights is $\frac{4}{6}$, and the ratio of the weight of A to the weight of B is $\frac{2}{6}$. Hence, the momentum of A after collision, is put down in the table as $\frac{4}{6}A + \frac{2}{6}B$. The meaning of the $\frac{4}{6}A$, is, that one part will be equal to four sixths of the momentum which A had before collision: that is, equal to four sixths of 10. The meaning of the $\frac{2}{6}B$ is, that the other part will be equal to two sixths of the momentum which B had before collision: that is, equal to two sixths of 60.

According to Law 3, B, after collision, will have a momentum, in the given direction of A, which,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of A to the weight of B; and a momentum in the given direction of B, which,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of A and B. In case No. 5, the

ratio is $\frac{2}{6}$ and the difference $\frac{4}{6}$. Hence, the momentum of B after collision, is put down in the table as $\frac{2}{6}A + \frac{4}{6}B$. The meaning of the $\frac{2}{6}A$, is, that one part will be equal to two sixths of the momentum which A had before collision: that is, equal to two sixths of 10. The meaning of the $\frac{4}{6}B$ is, that the other part will be equal to four sixths of the momentum which B had before collision: that is, equal to four sixths of 60.

According to Law 4, B, after collision, will have a velocity in the given direction of A, which,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B; and a velocity in the given direction of B, which,—as compared with the velocity of B before collision,—will be proportional to the difference between the respective weights of A and B. In case No. 5, the ratio is $\frac{2}{6}$, and the difference $\frac{4}{6}$. Hence, the velocity of B, after collision, is put down in the table as $\frac{2}{6}A^2 + \frac{4}{6}B$. The meaning of the $\frac{2}{6}A^2$, is, that the velocity of B, in the given direction of A, will be equal to four thirty-sixths of the velocity which A had before collision: that is, equal to four thirty-sixths of 5 feet per second. The meaning of the $\frac{4}{6}B$ is, that the velocity of B, in the given direction of B, will be equal to four sixths of the velocity which B had before collision: that is, equal to four sixths of 10 feet per second.

Now, whatever may be the weight of A, or the weight of B; and whatever may be the velocity of A, or the velocity of B, before collision; the Laws above enunciated will hold good in every case in which the weight of A is equal to, or less than, the weight of B; and in which B, at the time of collision, is moving in the same right line as that in which A is moving, but in an opposite direction.

TABLE.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Velocity of B before collision in feet per second.	Momentum of A before collision.	Momentum of B before collision.	Momentum of A and B before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	1	2	1	10	1	20	21		$\frac{1}{3}A+\frac{2}{3}B$	$\frac{1}{2}A+\frac{1}{3}B$	$\frac{1}{3}A+\frac{1}{3}B$	$\frac{1}{3}A^2+\frac{1}{3}B$
2	1	3	2	10	2	30	32		$\frac{2}{3}A+\frac{3}{3}B$	$\frac{2}{3}A+\frac{1}{3}B$	$\frac{1}{3}A+\frac{2}{3}B$	$\frac{1}{3}A^2+\frac{2}{3}B$
3	1	4	3	10	3	40	43		$\frac{3}{4}A+\frac{4}{4}B$	$\frac{3}{4}A+\frac{1}{4}B$	$\frac{1}{4}A+\frac{3}{4}B$	$\frac{1}{4}A^2+\frac{3}{4}B$
4	2	5	4	10	8	50	58		$\frac{3}{5}A+\frac{5}{5}B$	$\frac{3}{5}A+\frac{2}{5}B$	$\frac{2}{5}A+\frac{3}{5}B$	$\frac{2}{5}A^2+\frac{3}{5}B$
5	2	6	5	10	10	60	70		$\frac{4}{6}A+\frac{6}{6}B$	$\frac{4}{6}A+\frac{2}{6}B$	$\frac{2}{6}A+\frac{4}{6}B$	$\frac{2}{6}A^2+\frac{4}{6}B$
6	2	7	6	10	12	70	82		$\frac{5}{7}A+\frac{7}{7}B$	$\frac{5}{7}A+\frac{2}{7}B$	$\frac{2}{7}A+\frac{5}{7}B$	$\frac{2}{7}A^2+\frac{5}{7}B$
7	3	8	7	10	21	80	101		$\frac{5}{8}A+\frac{8}{8}B$	$\frac{5}{8}A+\frac{3}{8}B$	$\frac{3}{8}A+\frac{5}{8}B$	$\frac{3}{8}A^2+\frac{5}{8}B$
8	3	9	8	9	24	81	105		$\frac{6}{9}A+\frac{9}{9}B$	$\frac{6}{9}A+\frac{3}{9}B$	$\frac{3}{9}A+\frac{6}{9}B$	$\frac{3}{9}A^2+\frac{6}{9}B$
9	3	10	9	9	27	90	117		$\frac{7}{10}A+\frac{10}{10}B$	$\frac{7}{10}A+\frac{3}{10}B$	$\frac{3}{10}A+\frac{7}{10}B$	$\frac{3}{10}A^2+\frac{7}{10}B$
10	4	11	10	10	40	110	150		$\frac{7}{11}A+\frac{11}{11}B$	$\frac{7}{11}A+\frac{4}{11}B$	$\frac{4}{11}A+\frac{7}{11}B$	$\frac{4}{11}A^2+\frac{7}{11}B$
11	4	12	9	10	36	120	156		$\frac{8}{12}A+\frac{12}{12}B$	$\frac{8}{12}A+\frac{4}{12}B$	$\frac{4}{12}A+\frac{8}{12}B$	$\frac{4}{12}A^2+\frac{8}{12}B$
12	4	13	8	9	32	117	149		$\frac{9}{13}A+\frac{13}{13}B$	$\frac{9}{13}A+\frac{4}{13}B$	$\frac{4}{13}A+\frac{9}{13}B$	$\frac{4}{13}A^2+\frac{9}{13}B$

13	5	12	7	8	35	96	131		$\frac{7}{12}A+\frac{12}{12}B$	$\frac{7}{12}A+\frac{5}{12}B$	$\frac{5}{12}A+\frac{7}{12}B$	$\frac{5}{12}A^2+\frac{7}{12}B$
14	5	11	6	7	30	77	107		$\frac{6}{11}A+\frac{11}{11}B$	$\frac{6}{11}A+\frac{5}{11}B$	$\frac{5}{11}A+\frac{6}{11}B$	$\frac{5}{11}A^2+\frac{6}{11}B$
15	5	10	7	8	35	80	115		$\frac{5}{10}A+\frac{10}{10}B$	$\frac{5}{10}A+\frac{5}{10}B$	$\frac{5}{10}A+\frac{5}{10}B$	$\frac{5}{10}A^2+\frac{5}{10}B$
16	6	9	8	9	48	81	129		$\frac{3}{9}A+\frac{9}{9}B$	$\frac{3}{9}A+\frac{6}{9}B$	$\frac{6}{9}A+\frac{3}{9}B$	$\frac{6}{9}A^2+\frac{3}{9}B$
17	6	8	9	10	54	80	134		$\frac{2}{8}A+\frac{8}{8}B$	$\frac{2}{8}A+\frac{6}{8}B$	$\frac{6}{8}A+\frac{2}{8}B$	$\frac{6}{8}A^2+\frac{2}{8}B$
18	6	7	10	11	60	77	137		$\frac{1}{7}A+\frac{7}{7}B$	$\frac{1}{7}A+\frac{6}{7}B$	$\frac{6}{7}A+\frac{1}{7}B$	$\frac{6}{7}A^2+\frac{1}{7}B$
19	7	8	10	12	70	96	166		$\frac{1}{8}A+\frac{8}{8}B$	$\frac{1}{8}A+\frac{7}{8}B$	$\frac{7}{8}A+\frac{1}{8}B$	$\frac{7}{8}A^2+\frac{1}{8}B$
20	7	9	9	11	63	99	162		$\frac{2}{9}A+\frac{9}{9}B$	$\frac{2}{9}A+\frac{7}{9}B$	$\frac{7}{9}A+\frac{2}{9}B$	$\frac{7}{9}A^2+\frac{2}{9}B$
21	7	10	8	10	56	100	156		$\frac{3}{10}A+\frac{10}{10}B$	$\frac{3}{10}A+\frac{7}{10}B$	$\frac{7}{10}A+\frac{3}{10}B$	$\frac{7}{10}A^2+\frac{3}{10}B$
22	8	11	7	9	56	99	155		$\frac{3}{11}A+\frac{11}{11}B$	$\frac{3}{11}A+\frac{8}{11}B$	$\frac{8}{11}A+\frac{3}{11}B$	$\frac{8}{11}A^2+\frac{3}{11}B$
23	8	12	6	8	48	96	144		$\frac{4}{12}A+\frac{12}{12}B$	$\frac{4}{12}A+\frac{8}{12}B$	$\frac{8}{12}A+\frac{4}{12}B$	$\frac{8}{12}A^2+\frac{4}{12}B$
24	8	13	5	7	40	91	131		$\frac{5}{13}A+\frac{13}{13}B$	$\frac{5}{13}A+\frac{8}{13}B$	$\frac{8}{13}A+\frac{5}{13}B$	$\frac{8}{13}A^2+\frac{5}{13}B$
25	9	12	4	7	36	84	120		$\frac{3}{12}A+\frac{12}{12}B$	$\frac{3}{12}A+\frac{9}{12}B$	$\frac{9}{12}A+\frac{3}{12}B$	$\frac{9}{12}A^2+\frac{3}{12}B$
26	9	11	3	8	27	88	115		$\frac{2}{11}A+\frac{11}{11}B$	$\frac{2}{11}A+\frac{9}{11}B$	$\frac{9}{11}A+\frac{2}{11}B$	$\frac{9}{11}A^2+\frac{2}{11}B$
27	9	10	2	9	18	90	108		$\frac{1}{10}A+\frac{10}{10}B$	$\frac{1}{10}A+\frac{9}{10}B$	$\frac{9}{10}A+\frac{1}{10}B$	$\frac{9}{10}A^2+\frac{1}{10}B$
28	10	11	1	10	10	110	120		$\frac{1}{11}A+\frac{11}{11}B$	$\frac{1}{11}A+\frac{10}{11}B$	$\frac{10}{11}A+\frac{1}{11}B$	$\frac{10}{11}A^2+\frac{1}{11}B$
29	10	12	2	10	20	120	140		$\frac{2}{12}A+\frac{12}{12}B$	$\frac{2}{12}A+\frac{10}{12}B$	$\frac{10}{12}A+\frac{2}{12}B$	$\frac{10}{12}A^2+\frac{2}{12}B$
30	10	13	3	10	30	130	160		$\frac{3}{13}A+\frac{13}{13}B$	$\frac{3}{13}A+\frac{10}{13}B$	$\frac{10}{13}A+\frac{3}{13}B$	$\frac{10}{13}A^2+\frac{3}{13}B$

SECTION VIII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE EIGHTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF B; AND IN WHICH B, AT THE TIME OF COLLISION, IS MOVING IN THE SAME RIGHT LINE AS THAT IN WHICH A IS MOVING, BUT IN AN OPPOSITE DIRECTION.

In every case belonging to this class, the results of collision will be determined by the following laws :—

Law 1. A will have a velocity in its own given direction, which,—as compared with the given velocity of A before collision,—will be proportional to the difference between the respective weights of B and A ; and a velocity in the given direction of B, which,—as compared with the velocity of B before collision,—will be proportional to the square of the ratio of the weight of B to the weight of A.

Law 2. The momentum of A, in the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of B and A. The momentum of A, in the given direction of B,—as compared with the momentum of B before collision,—will be proportional to the ratio of the weight of B to the weight of A.

Law 3. The momentum of B will consist of two parts. One part,—as compared with the momentum of A before

collision,—will be proportional to the ratio of the weight of B to the weight of A. The other part,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of B and A.

Law 4. B will move in the given direction of A. The velocity of B will consist of two parts. One part will be equal to the velocity of A before collision; and the other part,—as compared with the velocity of B before collision,—will be proportional to the difference between the respective weights of B and A.

These Laws will give the results of collision in terms of the weight of A; and the results may all be expressed by four fractional numbers; namely, that which expresses the ratio of the weight of B to the weight of A; that which expresses the difference between the respective weights of B and A; that which expresses the sum of the ratio and difference of the weights; and that which expresses the square of the ratio of the weight of B to the weight of A.

In case No. 4, of the following Table, for example, the weight of A is 5 pounds, and the velocity of A before collision is four feet per second. Hence, the momentum of A before collision, is put down in the Table as 20. The weight of B is 2 pounds, and the velocity of B before collision 5 feet per second. Hence, the momentum of B, before collision, is put down in the Table as 10. And hence, the joint momentum of A and B is put down in the Table as 30: which is the amount of the momentum that is to be accounted for after collision, in case No. 4.

Now, according to Law 1, the velocity of A, after collision, in the given direction of A,—as compared with the velocity of A before collision,—will be proportional

to the difference between the respective weights of B and A; and the velocity of A in the given direction of B,—as compared with the velocity of B before collision,—will be proportional to the square of the ratio of the weight of B to the weight of A. In case No. 4, the difference between the weights is $\frac{3}{5}$, and the ratio of the weight of B to the weight of A is $\frac{2}{5}$. Hence, the velocity of A, after collision, is put down in the Table as $\frac{3}{5}A + \frac{2}{5}B^2$; the meaning of which is, that the velocity of A, in the given direction of A, will be equal to three fifths of the velocity which A had before collision; that is, equal to three fifths of 4 feet per second; and that the velocity of A, in the given direction of B, will be equal to four twenty-fifths of the velocity which B had before collision: that is, equal to four twenty-fifths of five feet per second.

According to Law 2, the momentum of A after collision, in the given direction of A,—as compared with the momentum of A before collision,—will be proportional to the difference between the respective weights of B and A; and the momentum of A in the given direction of B,—as compared with the momentum of B before collision,—will be proportional to the ratio of the weight of B to the weight of A. In case No. 4, the difference between the weights is $\frac{3}{5}$, and the ratio of the weight of B to the weight of A is $\frac{2}{5}$. Hence, the momentum of A is put down in the table as $\frac{3}{5}A + \frac{2}{5}B$; the meaning of which is, that the momentum of A, in the given direction of A, will be equal to three fifths of the momentum which A had before collision: that is, equal to three fifths of 20; and that the momentum of A, in the given direction of B, will be equal to two fifths of the momentum which B had before collision: that is, equal to two fifths of 10.

According to Law 3, the momentum of B will consist of two parts. One part,—as compared with the momentum of A before collision,—will be proportional to the ratio of the weight of B to the weight of A; and the other part,—as compared with the momentum of B before collision,—will be proportional to the difference between the respective weights of B and A. In case No. 4, the ratio of the weight of B to the weight of A is $\frac{2}{5}$; and the difference between the weights is $\frac{3}{5}$. Hence, the momentum of B after collision, is put down in the table as $\frac{2}{5}A + \frac{3}{5}B$; the meaning of which is, that one part will be equal to two fifths of the momentum which A had before collision: that is, equal to two fifths of 20; and that the other part will be equal to three fifths of the momentum which B had before collision: that is, equal to three fifths of 10.

According to Law 4, the velocity of B, after collision, will consist of two parts. One part will be equal to the velocity of A before collision; and the other part,—as compared with the velocity of B before collision,—will be proportional to the difference between the respective weights of B and A. In case No. 4, the velocity of A is $\frac{5}{5}$, and the difference between the weights is $\frac{3}{5}$. Hence, the velocity of B, after collision, is put down in the table as $\frac{5}{5}A + \frac{3}{5}B$; the meaning of which is, that one part will be equal to five fifths of the velocity which A had before collision: that is, equal to five fifths of 4 feet per second; and that the other part will be equal to three fifths of the velocity which B had before collision: that is, equal to three fifths of 5 feet per second.

Now, whatever may be the weight of A, or the weight of B; or whatever may be the velocity of either A or B before collision; the Laws above enunciated will

hold good in every case in which the weight of A is equal to, or greater than, the weight of B, and in which B, at the time of collision, is moving in the same right line as that in which A is moving, but in an opposite direction.

TABLE.

No. of the Case.	Weight of A in pounds.	Weight of B in pounds.	Velocity of A before collision in feet per second.	Velocity of B before collision in feet per second.	Momentum of A before collision.	Momentum of B before collision.	Momentum of A and B before collision.		Velocity of A after collision.	Momentum of A after collision.	Momentum of B after collision.	Velocity of B after collision.
1	2	1	1	2	2	2	4		$\frac{1}{2}A+\frac{1}{2}B^2$	$\frac{1}{2}A+\frac{1}{2}B$	$\frac{1}{2}A+\frac{1}{2}B$	$\frac{2}{2}A+\frac{1}{2}B$
2	3	1	2	3	6	3	9		$\frac{2}{3}A+\frac{1}{3}B^2$	$\frac{2}{3}A+\frac{1}{3}B$	$\frac{1}{3}A+\frac{2}{3}B$	$\frac{3}{3}A+\frac{2}{3}B$
3	4	1	3	4	12	4	16		$\frac{3}{4}A+\frac{1}{4}B^2$	$\frac{3}{4}A+\frac{1}{4}B$	$\frac{1}{4}A+\frac{3}{4}B$	$\frac{4}{4}A+\frac{3}{4}B$
4	5	2	4	5	20	10	30		$\frac{3}{5}A+\frac{2}{5}B^2$	$\frac{3}{5}A+\frac{2}{5}B$	$\frac{2}{5}A+\frac{3}{5}B$	$\frac{5}{5}A+\frac{3}{5}B$
5	6	2	5	6	30	12	42		$\frac{4}{6}A+\frac{2}{6}B^2$	$\frac{4}{6}A+\frac{2}{6}B$	$\frac{2}{6}A+\frac{4}{6}B$	$\frac{6}{6}A+\frac{4}{6}B$
6	7	2	6	7	42	14	56		$\frac{5}{7}A+\frac{2}{7}B^2$	$\frac{5}{7}A+\frac{2}{7}B$	$\frac{2}{7}A+\frac{5}{7}B$	$\frac{7}{7}A+\frac{5}{7}B$
7	8	3	7	8	56	24	80		$\frac{5}{8}A+\frac{3}{8}B^2$	$\frac{5}{8}A+\frac{3}{8}B$	$\frac{3}{8}A+\frac{5}{8}B$	$\frac{8}{8}A+\frac{5}{8}B$
8	9	3	8	9	72	27	99		$\frac{6}{9}A+\frac{3}{9}B^2$	$\frac{6}{9}A+\frac{3}{9}B$	$\frac{3}{9}A+\frac{6}{9}B$	$\frac{9}{9}A+\frac{6}{9}B$
9	10	3	9	10	90	30	120		$\frac{7}{10}A+\frac{3}{10}B^2$	$\frac{7}{10}A+\frac{3}{10}B$	$\frac{3}{10}A+\frac{7}{10}B$	$\frac{10}{10}A+\frac{7}{10}B$
10	11	4	10	11	110	44	154		$\frac{7}{11}A+\frac{4}{11}B^2$	$\frac{7}{11}A+\frac{4}{11}B$	$\frac{4}{11}A+\frac{7}{11}B$	$\frac{11}{11}A+\frac{7}{11}B$
11	12	4	11	12	132	48	180		$\frac{8}{12}A+\frac{4}{12}B^2$	$\frac{8}{12}A+\frac{4}{12}B$	$\frac{4}{12}A+\frac{8}{12}B$	$\frac{12}{12}A+\frac{8}{12}B$
12	13	4	12	13	156	52	208		$\frac{9}{13}A+\frac{4}{13}B^2$	$\frac{9}{13}A+\frac{4}{13}B$	$\frac{4}{13}A+\frac{9}{13}B$	$\frac{13}{13}A+\frac{9}{13}B$

13	14	5	11	14	154	70	224		$\frac{9}{14}A+\frac{5}{14}B^2$	$\frac{9}{14}A+\frac{5}{14}B$	$\frac{5}{14}A+\frac{9}{14}B$	$\frac{14}{14}A+\frac{9}{14}B$
14	15	5	10	15	150	75	225		$\frac{10}{15}A+\frac{5}{15}B^2$	$\frac{10}{15}A+\frac{5}{15}B$	$\frac{5}{15}A+\frac{10}{15}B$	$\frac{15}{15}A+\frac{10}{15}B$
15	16	5	9	16	144	80	224		$\frac{11}{16}A+\frac{5}{16}B^2$	$\frac{11}{16}A+\frac{5}{16}B$	$\frac{5}{16}A+\frac{11}{16}B$	$\frac{16}{16}A+\frac{11}{16}B$
16	17	6	8	15	136	90	226		$\frac{11}{17}A+\frac{6}{17}B^2$	$\frac{11}{17}A+\frac{6}{17}B$	$\frac{6}{17}A+\frac{11}{17}B$	$\frac{17}{17}A+\frac{11}{17}B$
17	18	6	7	14	126	84	210		$\frac{12}{18}A+\frac{6}{18}B^2$	$\frac{12}{18}A+\frac{6}{18}B$	$\frac{6}{18}A+\frac{12}{18}B$	$\frac{18}{18}A+\frac{12}{18}B$
18	19	6	6	13	114	78	192		$\frac{13}{19}A+\frac{6}{19}B^2$	$\frac{13}{19}A+\frac{6}{19}B$	$\frac{6}{19}A+\frac{13}{19}B$	$\frac{19}{19}A+\frac{13}{19}B$
19	20	7	5	12	100	84	184		$\frac{13}{20}A+\frac{7}{20}B^2$	$\frac{13}{20}A+\frac{7}{20}B$	$\frac{7}{20}A+\frac{13}{20}B$	$\frac{20}{20}A+\frac{13}{20}B$
20	19	7	4	11	76	77	153		$\frac{12}{19}A+\frac{7}{19}B^2$	$\frac{12}{19}A+\frac{7}{19}B$	$\frac{7}{19}A+\frac{12}{19}B$	$\frac{19}{19}A+\frac{12}{19}B$
21	18	7	3	10	54	70	124		$\frac{11}{18}A+\frac{7}{18}B^2$	$\frac{11}{18}A+\frac{7}{18}B$	$\frac{7}{18}A+\frac{11}{18}B$	$\frac{18}{18}A+\frac{11}{18}B$
22	17	8	2	9	34	72	106.		$\frac{9}{17}A+\frac{8}{17}B^2$	$\frac{9}{17}A+\frac{8}{17}B$	$\frac{8}{17}A+\frac{9}{17}B$	$\frac{17}{17}A+\frac{9}{17}B$
23	16	8	1	8	16	64	80		$\frac{8}{16}A+\frac{8}{16}B^2$	$\frac{8}{16}A+\frac{8}{16}B$	$\frac{8}{16}A+\frac{8}{16}B$	$\frac{16}{16}A+\frac{8}{16}B$
24	15	8	2	7	30	56	86		$\frac{7}{15}A+\frac{8}{15}B^2$	$\frac{7}{15}A+\frac{8}{15}B$	$\frac{8}{15}A+\frac{7}{15}B$	$\frac{15}{15}A+\frac{7}{15}B$
25	14	9	3	6	42	54	96		$\frac{5}{14}A+\frac{9}{14}B^2$	$\frac{5}{14}A+\frac{9}{14}B$	$\frac{9}{14}A+\frac{5}{14}B$	$\frac{14}{14}A+\frac{5}{14}B$
26	13	9	4	5	52	45	97		$\frac{4}{13}A+\frac{9}{13}B^2$	$\frac{4}{13}A+\frac{9}{13}B$	$\frac{9}{13}A+\frac{4}{13}B$	$\frac{13}{13}A+\frac{4}{13}B$
27	12	9	5	7	60	63	123		$\frac{3}{12}A+\frac{9}{12}B^2$	$\frac{3}{12}A+\frac{9}{12}B$	$\frac{9}{12}A+\frac{3}{12}B$	$\frac{12}{12}A+\frac{3}{12}B$
28	11	10	6	9	66	90	156		$\frac{1}{11}A+\frac{10}{11}B^2$	$\frac{1}{11}A+\frac{10}{11}B$	$\frac{10}{11}A+\frac{1}{11}B$	$\frac{11}{11}A+\frac{1}{11}B$
29	10	9	7	10	70	90	160		$\frac{1}{10}A+\frac{9}{10}B^2$	$\frac{1}{10}A+\frac{9}{10}B$	$\frac{9}{10}A+\frac{1}{10}B$	$\frac{10}{10}A+\frac{1}{10}B$
30	9	8	8	12	72	96	168		$\frac{1}{9}A+\frac{8}{9}B^2$	$\frac{1}{9}A+\frac{8}{9}B$	$\frac{8}{9}A+\frac{1}{9}B$	$\frac{9}{9}A+\frac{1}{9}B$

SECTION IX.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE NINTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR LESS THAN, THE WEIGHT OF C; IN WHICH THE WEIGHT OF C IS EQUAL TO, OR LESS THAN, THE WEIGHT OF B; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH A AND C MOVE IN THE SAME RIGHT LINE,—BUT IN OPPOSITE DIRECTIONS,—WITH ANY VELOCITY WHATEVER; AND IN WHICH BOTH A AND C COME IN COLLISION WITH B AT THE SAME INSTANT OF TIME.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity of A will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the difference between the respective weights of A and C; and the other part will be equal to the velocity of C before collision.

Law 2. C will be reflected back, from the point of collision, in the same right line that it moved in before collision. One part,—as compared with the velocity of C before collision,—will be proportional to the difference between the respective weights of C and B, minus a quantity proportional to the ratio of the weight of A to the weight of C. The other part,—as compared with the

velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of C.

Law 3. B will move in the given direction of C. The velocity of B,—as compared with the velocity of C before collision,—will be proportional to the square of the ratio of the weight of C to the weight of B, minus a quantity proportional to the ratio of the weight of B to the weight of C.

SECTION X.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE TENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF B; IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH A AND C MOVE IN THE SAME RIGHT LINE,—BUT IN OPPOSITE DIRECTIONS,—WITH ANY VELOCITY WHATEVER; AND IN WHICH BOTH A AND C COME IN COLLISION WITH B AT THE SAME INSTANT OF TIME.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity of A will consist of two parts. One part,—as compared with the velocity of A before collision,—will be proportional to the difference between the re-

spective weights of A and C. The other part will be equal to the velocity of C before collision.

LAW 2. C will have a velocity in its own given direction, and also a velocity in the given direction of A. The velocity of C in its own given direction,—as compared with the velocity of C before collision,—will be proportional to the difference between the respective weights of A and C, minus a quantity proportional to the ratio of the weight of B to the weight of C. The velocity of C in the given direction of A,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of C.

LAW 3. B will move in the given direction of C. The velocity of B,—as compared with the velocity of C before collision,—will be proportional to the difference between the respective weights of A and C.

SECTION XI.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE ELEVENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS LESS THAN THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE LINES OF DIRECTION IN WHICH A AND B RESPECTIVELY MOVE, BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 0 AND 90 DEGREES; AND IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH, THAT THE LINE OF DIRECTION IN WHICH A MOVES WILL PASS THROUGH THE CENTER OF GRAVITY OF B.

In this,—and also in the following sections,—the results of collision will be expressed in terms of the sine, cosine, &c., of the angle formed by the lines of direction in which A and B respectively move before collision; and as the amount of the momentum, in any given case, can always be determined by multiplying the weight by the velocity of the body, those Laws only will be enunciated which determine the directions and the respective velocities of A and B after collision.

Let A′E and F D (Fig. 1, Plate 1) be two diameters, at right-angles to each other, of the circle A′D E F; and let C, the center of the circle, be the point of collision in all cases. Suppose that A, before collision, will in every case describe the radius A′C, in one second of time; and that B, in a given case, will describe the radius B′C, in

the same time. Then A′ C B′ will be the angle formed by the lines of direction of A and B; and the respective velocities of A and B before collision will be equal to radius, and therefore equal to each other.

Now, in every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will have a velocity, in its own given direction, that will be proportional to the cosine of the angle; and a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of A and B, minus a quantity proportional to the ratio of the cosine to radius. The line that will be actually described by A, after collision, in the given time, will be proportional to the difference between the two opposite velocities aforesaid.

LAW 2. B will have a velocity, in its own given direction, that will be equal to radius; and a velocity in the given direction of A, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of A to the weight of B, minus a quantity proportional to the ratio of the cosine to radius. The line that will be actually described by B, after collision, in the given time, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

Now, in order to find the respective positions of A and B at the end of one second after collision, suppose that A will describe the radius A′ C (Fig. 1, Plate 1), and B the radius B′ C, in one second before collision. Then A′ C B′ will be the angle formed by the lines of direction in which A and B respectively move before collision; B′*s* will be the sine, and *s* C the cosine of that angle. Suppose, also, that the weight of A is equal to 1 pound;

that the weight of B is equal to 2 pounds; and that the cosine of the angle is equal to two thirds of radius. Now, in order to find the position of A at the end of one second after collision, we must look to Law 1, which says that the velocity of A, in the direction of the radius C E, will be proportional to the cosine of the angle. Hence, if we make C a' equal to the cosine C b, the point a' will be the position of A, so far as the velocity of A in the direction C E is concerned. According to the same Law, the velocity of A in the opposite direction, will be proportional to one half of radius, minus two thirds of one half of radius: which, in this case, will make the velocity of A, in the direction of the radius C A′, proportional to one sixth of radius. Hence, if we make $a'a$ equal to one sixth of radius, the point a, in the radius C E, will be the position of A at the end of one second after collision: having described, in that time, the line C $a'a$.

In order to find the position of B, at the end of one second after collision, we must look to Law 2, which says that the velocity of B after collision, in its own given direction, will be proportional to radius. Hence, if we produce B′C to g, touching the circumference in g, the point g will be the position of B, so far as the velocity of B in the given direction of B is concerned. But the same Law says that the velocity of B, in the given direction of A, will, in this case, be proportional to one fourth of radius, minus two thirds of one fourth of radius. This would make the velocity of B, in this case, in the given direction of A, proportional to one twelfth of radius. Hence, if we draw $g b$, parallel to C E, and make it equal to one twelfth of radius, the line $g b$ will represent the velocity of B in the given direction of A. From the two lines C g and $g b$, complete the

parallelogram, and draw the diagonal C *b*. Then the point *b* will be the position of B, at the end of one second after collision: having described, in that time, the diagonal C *b*.

Now C *a′*, plus *a′ a*, multiplied by the weight of A, added to C *g*, plus *g b*, multiplied by the weight of B; will be equal to A′C multiplied by the weight of A, added to B′C multiplied by the weight of B: that is, the joint momentum of A and B after collision, will be equal to the joint momentum of A and B before collision.

SECTION XII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE TWELFTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS LESS THAN THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH THAT THE LINE OF DIRECTION IN WHICH A IS MOVING WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND B MOVE BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES.

IN every case belonging to this class, the results of collision will be determined by the following laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision.

The velocity of A will consist of two parts. One part,—as compared with radius,—will be proportional to the difference between the respective weights of A and B. The other part will be proportional to the cosine of the angle.

LAW 2. B will have a velocity in the given direction of A, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of A to the weight of B. B will also have a velocity in its own given direction, that will be proportional to radius, minus a quantity,—as compared with the cosine of the angle,—proportional to the ratio of the weight of A to the weight of B.

For example, let the difference between the respective weights of A and B be equal to $\frac{1}{2}$; and suppose that A will describe the radius $A'C$, and B the radius $B'C$ (Fig. 2, Plate 1), in one second before collision. Then $A'CB'$ will be the angle formed by the lines of direction in which A and B respectively move before collision; $B's$ will be the sine; $C's$ the cosine; and Es the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will move in the direction of the radius CA'; that one part of its velocity,—as compared with radius,—will be proportional to the difference between the respective weights of A and B. That difference, in this case, is equal to $\frac{1}{2}$. Hence, if we make Ca' equal to one half of radius, the line Ca' will represent one part of the velocity of A. According to the same Law, the other part will be proportional to the cosine of the angle. Hence, if we make $a'a$ equal to the cosine Cs, the line $a'a$ will represent the other part of the velocity of A; and the point a will be the position of A, at the end of one second

after collision: having described, in that time, the right line Ca' a.

To find the position of B, at the end of one second after collision, we must look to Law 2, which says that B will have a velocity in the given direction of A, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of A to the weight of B. That square, in this case, is equal to $\frac{1}{4}$. Hence, if we make C b' equal to one fourth of radius, the line C b' will represent the velocity of B in the given direction of A. According to the same law, B will have a velocity in its own given direction, that will be proportional to radius, minus a quantity,—as compared with the cosine of the angle,—proportional to the ratio of the weight of A to the weight of B. That ratio, in this case, is equal to $\frac{1}{2}$. Hence, if we make C b'' equal to radius, minus one half of the cosine C b, the line C b'' will represent the velocity of B, in its own given direction. From the lines C b' and C b'', complete the parallelogram, and draw the diagonal C b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal C b.

Now, C a', plus $a'a$, multiplied by the weight of A, added to C b', plus C b'', multiplied by the weight of B; will be equal to A$'$C multiplied by the weight of A, added to B$'$C multiplied by the weight of B: that is, the joint momentum of A and B, after collision, will be equal to the joint momentum of A and B before collision.

SECTION XIII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE THIRTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH THAT THE LINE OF DIRECTION IN WHICH A MOVES WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE LINES OF DIRECTION IN WHICH A AND B RESPECTIVELY MOVE BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 0 AND 90 DEGREES, INCLUSIVE.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will continue to move, in its own given direction, with a velocity that will be proportional to the cosine of the angle.

Law 2. B will have a velocity, in its own given direction, that will be proportional to radius; and a velocity, in the given direction of A, that will be proportional to the versed sine of the angle. The line that will be actually described by B, after collision, in the given time, will coincide with the diagonal of a parallelogram completed from the lines which represent the directions and velocities aforesaid.

Suppose, for example, that A, during one second before collision, would describe the radius A′C, while

B described the radius B′C (Fig. 1, Plate 2). Then A′C B′ would be the angle formed by the lines of direction; B′*b* would be the sine; *b* C the cosine; and A *b* the versed sine, of that angle. From the extremity E, of the radius C E, draw the right line E *t*, equal and parallel to the radius C F. This line will be a tangent in E.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will move in the direction of the radius C E, with a velocity that will be proportional to the cosine of the angle. Hence, if we make C *a* equal to the cosine C *b*, the point *a*, in the radius C E, will be the position of A, at the end of one second after collision: having described, in that time, the right line C *a*.

In order to find the position of B, at the end of one second after collision, we must look to Law 2, which says that B will have a velocity, in its own given direction, that will be proportional to radius. Hence, if we produce B′C to *v*, touching the circumference in *v*, the point *v* will be the position of B, so far as the velocity of B in its own given direction is concerned. According to the same law, B will have a velocity in the given direction of A, that will be proportional to the versed sine of the angle. Hence, if we draw *v b*, parallel to C E, and make it equal to the versed sine A′*b*, the line *v b* will represent the velocity of B in the given direction of A. From the lines C *v* and *v b* complete the parallelogram, and draw the diagonal C *b*. Then the point *b* will be the position of B at the end of one second after collision: having described, in that time, the diagonal C *b*.

Now, C *a* multiplied by the weight of A, added to C *v*, plus *v b*, multiplied by the weight of B; will be equal to A′C multiplied by the weight of A, added to

B′C multiplied by the weight of B: that is, the joint momentum of A and B after collision, will be equal to the joint momentum of A and B before collision.

It follows from the laws above enunciated that B, at the end of one second after collision, will always be found in the tangent Et, and that its distance from the point E will be equal to the sine of the angle.

SECTION XIV.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE FOURTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH THAT THE LINE OF DIRECTION IN WHICH A IS MOVING WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE LINES OF DIRECTION IN WHICH A AND B RESPECTIVELY MOVE, BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision, with a velocity proportional to the cosine of the angle.

LAW 2. B will have a velocity in two directions; namely, a velocity in a reflected direction, such, that the

angle of reflection will be equal to the angle of B's incidence upon A; and a velocity in the given direction of A. The velocity of B, in its reflected direction, will be proportional to radius; and its velocity in the given direction of A, will be proportional to the versed sine of the angle. The line that will be actually described by B, after collision, in the given time, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, suppose that A describes the radius A′C, and B the radius B′C (Fig. 2, Plate 2), in one second of time before collision. Then A′CB′ will be the angle formed by the lines of direction; B′*b* will be the sine; C*b* the cosine; and E*b* the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will be reflected back, from the point of collision, in the direction of the radius CA′; and that its velocity will be proportional to the cosine of the angle. Hence, if we make C*a* equal to the cosine C*b*, the point *a*, in the radius CA′, will be the position of A, at the end of one second after collision: having described, in that time, the line C*a*.

To find the position of B, at the end of one second after collision, we must look to Law 2, which says that B will have a velocity in a reflected direction, such, that the angle of reflection will be equal to the angle of B's incidence upon A; and that it will be proportional to radius. Hence, if we draw the radius C*v* in such a direction that the angle *v*CE will be equal to the angle B′CE, the radius C*v* will represent the direction in which B will have a velocity that will be equal to C*v*. According to the same Law, B will have a velocity, in the given direction of A, that will be proportional to the versed

sine of the angle. Hence, if we draw $v b$ parallel to C E, and make it equal to the versed sine E s, the line $v b$ will represent the velocity of B in the given direction of A. From the two lines $c v$ and $v b$, complete the parallelogram, and draw the diagonal C b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal C b.

Now, C a multiplied by the weight of A, added to C v, plus $v b$, multiplied by the weight of B; will be eqnal to A′C multiplied by the weight of A, added to B′C multiplied by the weight of B: that is, the joint momentum of A and B after collision, will be equal to the joint momentum of A and B before collision.

It follows from the Laws above enunciated that B, at the end of one second after collision, will always be found in the tangent E t, and that its distance from the point E will be equal to the sine of the angle.

By comparing the Laws of this with those of the preceding section, it will be seen that at equal angles above and below 90 degrees,—as at 45 and 135, for example, B will describe the same line precisely.

COROLLARY TO THE LAWS OF SECTIONS XIII. AND XIV.

Let the right lines A′E and F D be two diameters, at right angles to each other, of the circle A′D E F (Fig. 1, Plate 3). At the distance of radius from the diameter F D, in the direction C E, draw the right line t E v, equal and parallel to F D. Let C, the center of the circle, be the point of collision, and suppose that A, in every case, will describe the radius A′C in one second before collision. Suppose B to be carried, in the direction A′D, through the whole circumference of the circle, and at each point be made to move toward C, with a velocity equal to that

of A, and at such times that A will always come in collision with B, at the center C. Now A, at the end of one second after collision, will always be found somewhere in the diameter A′E, and B will be found somewhere in the right line tEv, which is perpendicular to A′E. When the angle is equal to 0, A will be found at the point E, in the extremity of the diameter AE; and while the angle is being increased from 0 to 90 degrees, A will be found nearer and nearer to C; and when the angle is equal to 90 degrees, A will be found at C. While the angle is being increased from 90 to 180 degrees, A will approach nearer and nearer to A′; and when the angle becomes equal to 180 degrees, A will be found at A′. While the angle is being increased from 180 to 270 degrees, A will return toward C; and when the angle becomes equal to 270 degrees, A will be found at C. While the angle is being increased from 270 to 360 degrees, A will approach nearer and nearer to E, and when the angle becomes equal to 360 degrees, A will be again found at E. In corresponding times, when the angle is equal to 0, B will be found at the point E; and while the angle is being increased from 0 to 90 degrees, B will approach nearer and nearer to the point t, and when the angle becomes equal to 90 degrees, B will found at t. While the angle is being increased from 90 to 180 degrees, B will return toward E, and when the angle becomes equal to 180 degrees, B will again be found at E. While the angle is being increased from 180 to 270 degrees, B will approach nearer and nearer to v, and when the angle becomes equal to 270 degrees, B will be found at v. While the angle is being increased from 270 to 360 degrees, B will return toward E; and when the angle becomes equal to 360 degrees, B will again be found at E. The distance of A from the center C, at the end of one second

after collision, will always be equal to the cosine of the angle; and the distance of B, at the same instant, from the point E, will always be equal to the sine of the angle.

SECTION XV.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE FIFTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS GREATER THAN THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH, THAT THE LINE OF DIRECTION IN WHICH A IS MOVING WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE LINES OF DIRECTION IN WHICH A AND B RESPECTIVELY MOVE BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATSOEVER BETWEEN 0 AND 90 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will continue to move in its own given direction. The velocity of A will consist of two parts. One part,—as compared with radius,—will be proportional to the difference between the respective weights of A and B. The other part,—as compared with the cosine of the angle,—will be proportional to the ratio of the weight of B to the weight of A.

LAW 2. B will have a velocity in two directions;

namely, a velocity in its own given direction,—that will be proportional to radius,—and a velocity in the given direction of A, that will be proportional to the versed sine of the angle. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, let the weight of A be equal to 2 pounds, and the weight of B equal to 1 pound; and suppose that A will describe the radius A′C, and B the radius B′C (Fig. 2, Plate 3), in one second before collision. Then A′C B′ will be the angle formed by the lines of direction; B′*b* will be the sine; C *b* the cosine; and A′*b* the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will move in the direction of the radius C E; and that one part of its velocity,—as compared with radius,—will be proportional to the difference between the respective weights of A and B. That difference, in this case, is equal to ½. Hence, if we make C*a*′ equal to one half of radius, the line C*a*′ will represent one part of the velocity of A. According to the same Law, the other part,—as compared with the cosine of the angle,—will be proportional to the ratio of the weight of B to the weight of A: which, in this case, is equal to ½. Hence, if we make *a*′*a* equal to one half of the cosine C*b*, the point *a* will be the position of A, at the end of one second after collision: having described, in that time, the right line C*a*′*a*.

In order to find the position of B, at the end of one second after collision, we must look to Law 2, which says that B will have a velocity, in its own given direc-

tion, that will be proportional to radius. Hence, if we produce the radius B′C to *v*, the radius C *v* will represent the velocity of B in its own direction. According to the same Law, B will have a velocity, in the given direction of A, that will be proportional to the versed sine of the angle. Hence, if we draw *v b*, parallel to C E, and make it equal to the versed sine A′*s*, the line *v b* will represent the velocity of B in the given direction of A. From the lines C *v* and *v b*, complete the parallelogram, and draw the diagonal C *b*. Then the point *b* will be the position of B, at the end of one second after collision: having described, in that time, the diagonal C *b*.

Now, C *a′*, plus *a′a*, multiplied by the weight of A, added to C *v*, plus *v b*, multiplied by the weight of B; will be equal to A′C multiplied by the weight of A, added to B′C multiplied by the weight of B: that is, the joint momentum of A and B after collision, will be equal to the joint momentum of A and B before collision.

It follows, from the Laws above enunciated, that B, at the end of one second after collision, will always be found in the tangent E *t*, and the distance of B from the point E, at that time, will always be equal to the sine of the angle.

SECTION XVI.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE SIXTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS GREATER THAN THE WEIGHT OF B; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF B; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS IN FRONT OF A, AND SUCH THAT THE LINE OF A'S DIRECTION WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE LINES OF DIRECTION IN WHICH A AND B RESPECTIVELY MOVE BEFORE COLLISION, MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of A and B. A will also have a velocity in opposition to its own given direction, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of B to the weight of A.

LAW 2. B will have a velocity in two directions; namely, a velocity in a reflected direction, such, that the angle of reflection will be equal to the angle of B's incidence upon A; and a velocity in the given direction of A. The velocity of B, in the reflected direction, will be proportional to radius. The velocity of B in the given direc-

tion of A,—as compared with the cosine of the angle,—will be proportional to the difference between the respective weights of A and B, plus a quantity proportional to the versed sine of the angle. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, let the difference between the weights be equal to $\frac{1}{2}$, and suppose that A will describe the radius A′C, and B the radius B′C (Fig. 1, Plate 4), in one second before collision. Then A′CB′ will be the angle formed by the lines of direction; B′*b* will be the sine; C*b* the cosine; and E*f* the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of A and B: which, in this case, is equal to $\frac{1}{2}$. Hence, if we make C*a*′ equal to one half of radius, the line C*a*′ will represent the velocity of A, in its own given direction. According to the same Law, A will have a velocity in the opposite direction, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of B to the weight of A: which, in this case, is equal to $\frac{1}{4}$. Hence, if we make *a*′*a* equal to one fourth of the cosine C*b*, the point *a* will be the position of A, at the end of one second after collision: having described, in that time, the right line C*a*.

In order to find the position of B, at the end of one second after collision, we must look to Law 2, which says that B will have a velocity, that will be proportional to

radius, in a reflected direction, such, that the angle of reflection will be equal to the angle of B's incidence upon A. Hence, if we draw the radius C v in such a direction that the angle v C E will be equal to the angle B′ C E, the radius C v will represent the direction in which B will have a velocity that will be equal to C v. According to the same Law, B will have a velocity in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the difference between the respective weights of A and B, plus a quantity proportional to the versed sine of the angle. The difference between the weights, in this case, is equal to ½. Hence, if we draw $v\,b$, parallel to C E, and make it equal to one half of the cosine C s, plus the versed sine E s, the line $v\,b$ will represent the velocity of B in the given direction of A. From the lines C v and $v\,b$, complete the parallelogram, and draw the diagonal C b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal C b.

Now, C a', plus $a'a$, multiplied by the weight of A, added to C v, plus $v\,b$, multiplied by the weight of B; will be equal to A′C, multiplied by the weight of A, added to B′C, multiplied by the weight of B: that is, the joint momentum of A and B after collision, will be equal to the joint momentum of A and B before collision.

SECTION XVII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE SEVENTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF B IS EQUAL TO, OR GREATER THAN, THE JOINT WEIGHT OF A AND C; IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH A AND C COME INTO COLLISION WITH B AT THE SAME INSTANT OF TIME; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATSOEVER BETWEEN 0 AND 90 DEGREES, INCLUSIVE.

In every case belonging to this class, the results of collision will be determined by the following laws:

LAW 1. A will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity of A,—as compared with radius,—will be proportional to the difference between the joint weight of A and C and the weight of B, plus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the joint weight of A and C and the weight of B, and the difference between the weight of A and the weight of B.

LAW 2. C will be reflected back, from the point of collision, in the same line that it moved in before collision. The velocity of C,—as compared with radius,—will be proportional to the difference between the joint weight of A and C and the weight of B, plus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the joint weight of A and C and the weight of B, and the difference between the weight of and C the weight of B.

LAW 3. B will have a velocity in the given direction of A, which,—as compared with radius,—will be proportional to $\frac{r}{a}$ of the square of the ratio of the joint weight of A and C to the weight of B, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between the square of the ratio of the weight of A to the weight of B, and $\frac{r}{a}$ of the square of the ratio of the joint weight of A and C to the weight of B: in which equation $\frac{r}{a}$ represents the ratio of the weight of A to the joint weight of A and C. B will also have a velocity in the given direction of C, which,—as compared with radius,—will be proportional to $\frac{r}{c}$ of the square of the ratio of the joint weight of A and C to the weight of B, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between the square of the ratio of the weight of C to the weight of B, and $\frac{r}{c}$ of the square of the ratio of the joint weight of A and C to the weight of B; in which equation $\frac{r}{c}$ represents the ratio of the weight of C to the joint weight of A and C. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed

from the two lines which represent the directions and velocities aforesaid.

For example, suppose that A will describe the radius A R, and C the radius C R (Fig. 2, Plate 4), in one second before collision. Then A R C will be the angle formed by the lines of direction of A and C; C b will be the sine; R b the cosine; and A b the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will be found in the radius A R; that the velocity of A,—as compared with radius,—will be proportional to the difference between the joint weight of A and C and the weight of B, plus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the joint weight of A and C and the weight of B, and the difference between the weight of A and the weight of B. Suppose the weight of A to be equal to 2 pounds; the weight of C 3 pounds; and the weight of B 6 pounds. Then the difference between the joint weight of A and C and the weight of B will be equal to $\frac{1}{6}$; the difference between the respective weights of A and B will be equal to $\frac{4}{6}$; and the difference between $\frac{1}{6}$ and $\frac{4}{6}$ is $\frac{3}{6}$. Hence, if we make R a equal to one sixth of radius, plus three sixths of the versed sine A b, the line R a will represent the velocity of A, and the point a will be the position of A, at the end of one second after collision.

To find the position of C, at the end of one second after collision, we must look to Law 2, which says that C will move in the direction of the radius R C; that the velocity of C,—as compared with radius,—will be proportional to the difference between the joint weight of A and C and

the weight of B, plus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the joint weight of A and C and the weight of B,—which, in this case, is equal to $\frac{1}{6}$,—and the difference between the weight of C and the weight of B,—which, in this case, is equal to $\frac{1}{2}$. The difference between $\frac{1}{6}$ and $\frac{1}{2}$ is equal to $\frac{1}{3}$. Hence, if we make R c equal to one sixth of radius, plus $\frac{1}{3}$ of the versed sine A b, the line R c will represent the velocity of C, and the point c will be the position of C, at the end of one second after collision.

To find the position of B, at the end of one second after collision, we must look to Law 3, which says that B will have a velocity in the given direction of A, which,—as compared with radius,—will be proportional to $\frac{r}{a}$ ($\frac{2}{5}$) of the square of the ratio of the joint weight of A and C to the weight of B ($\frac{25}{36}$), minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between the square of the ratio of the weight of A to the weight of B ($\frac{4}{36}$), and $\frac{r}{a}$ ($\frac{2}{5}$) of the square of the ratio of the joint weight of A and C to the weight of B: in which equation $\frac{r}{a}$ represents the ratio of the weight of A to the joint weight of A and C. Now, $\frac{2}{5}$ of $\frac{25}{36}$ is equal to $\frac{10}{36}$, and the difference between $\frac{10}{36}$ and $\frac{4}{36}$ is equal to $\frac{6}{36}$, or $\frac{1}{6}$. Hence, if we make R b' equal to ten thirty-sixths of radius, minus one sixth of the versed sine A b, the line R b' will represent the velocity of B in the given direction of A. The same Law says that B will have a velocity in the given direction of C, which,—as compared with radius,—will be proportional to $\frac{r}{c}$ (in this case equal to $\frac{3}{5}$) of the square of the ratio of the joint weight of A and C to the weight of B ($\frac{25}{36}$), minus a quantity,—as compared with the versed sine of the angle,—proportional to

the difference between the square of the ratio of the weight of C to the weight of B ($\frac{9}{36}$), and $\frac{r}{c}$ ($\frac{3}{5}$) of the square of the ratio of the joint weight of A and C to the weight of B: in which equation $\frac{r}{c}$ represents the ratio of the weight of C to the joint weight of A and C. Now, $\frac{3}{5}$ of $\frac{25}{36}$ is equal to $\frac{15}{36}$, and the difference between $\frac{15}{36}$ and $\frac{9}{36}$ is equal to $\frac{6}{36}$, or $\frac{1}{6}$. Hence, if we make R b'' equal to fifteen thirty-sixths of radius, minus one sixth of the versed sine A b, the line R b'' will represent the velocity of B in the given direction of C. From the lines R b' and R b'', complete the parallelogram, and draw the diagonal R b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal R b.

Now, R a multiplied by the weight of A, added to R c multiplied by the weight of C, added to R b', plus R b'', multiplied by the weight of B; will be equal to A R multiplied by the weight of A, added to C R multiplied by the weight of C: that is, the joint momentum of A, B, and C, after collision, will be equal to the joint momentum of A and C before collision.

If, in the example above given, we suppose radius to be equal to 10 feet, the respective velocities of A and C, before collision, would each be equal to 10 feet per second. And, as A is supposed to weigh 2 pounds, the momentum of A before collision, would be equal to 20. As the weight of C is supposed to be 3 pounds, the momentum of C before collision would be equal to 30. Hence, the joint momentum of A and C before collision would be equal to 50: which is the amount that is to be accounted for after collision. Now, if we suppose the angle to be such that the versed sine would be equal to 6 feet, the respective velocities and momenta of A, C, and B, after collision, would be as follows: The velocity

of A would be equal to $4\frac{2}{3}$ feet, and the momentum of A would be equal to $9\frac{1}{3}$. The velocity of C would be equal to $3\frac{2}{3}$ feet, and the momentum of C would be equal to 11. The velocity of B, in the given direction of A, would be equal to $1\frac{7}{9}$ feet, and the momentum of B equal to $10\frac{2}{3}$. The velocity of B, in the given direction of C, would be equal to $3\frac{1}{6}$ feet, and the momentum of B would be equal to 19. And $9\frac{1}{3} + 11 + 10\frac{2}{3} + 19 = 50$.

SECTION XVIII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE EIGHTEENTH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF B IS EQUAL TO, OR GREATER THAN, THE JOINT WEIGHT OF A AND C; IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH A AND C COME IN COLLISION WITH B AT THE SAME INSTANT OF TIME: IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will have a velocity in opposition to its own given direction, which,—as compared with radius,—will

be proportional to the difference between the respective weights of A and B, minus a quantity,—as compared with the cosine of the angle,—proportional to the difference between two other differences; namely, the difference between the respective weights of A and B, and the difference between the respective weights of A and C. A will also have a velocity, in the given direction of C, that will be proportional to the cosine of the angle. The line that will be actually described by A, after collision, in the given time, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

Law 2. C will have a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of C and B, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which equation $\frac{r}{c}$ represents the ratio of the cosine to radius. C will also have a velocity in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of C. The line that will be actually described by C, in the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

Law 3. B will have a velocity in the given direction of A, which,—as compared with the versed sine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of B. B will also have a velocity in the given direction of C, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of C to the weight of B, minus $\frac{r}{c}$

of a quantity proportional to the ratio of the weight of A to the weight of C: in which equation $\frac{r}{c}$ represents the ratio of the cosine to radius. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, let the weight of A be equal to 2 pounds; the weight of B equal to 6 pounds; and the weight of C equal to 3 pounds. And suppose that A, during one second before collision, will describe the radius A R, and C the radius C R (Fig. 1, Plate 5). Then A R C will be the angle formed by the lines of direction of A and C; C *s* will be the sine; R *s* the cosine; and E *s* the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will have a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of A and B, minus a quantity,—as compared with the cosine of the angle,—proportional to the difference between two other differences; namely, the difference between the respective weights of A and B, and the difference between the respective weights of A and C. As the difference, in this case, between the weights of A and B is equal to $\frac{2}{3}$, and the difference between the weights of A and C $\frac{1}{3}$, the difference between these two differences is equal to $\frac{1}{3}$. Hence, if we make R a' equal to two thirds of radius, minus one third of the cosine R *s*, the line R a' will represent the velocity of A, in opposition to its own given direction, during one second after collision. According to the same Law, A will have a velocity, in the given direction of C, that will be pro-

portional to the cosine of the angle. Hence, if we make R a'' equal to the cosine R s, the line R a'' will represent the velocity of A, in the given direction of C, during one second after collision. From the two lines R a' and R a'', complete the parallelogram, and draw the diagonal R a. Then the point a will be the position of A, at the end of one second after collision: having described, in that time, the diagonal R a.

In order to find the position of C, at the end of one second after collision, we must look to Law 2, which says that C will have a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of C and B, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C; in which $\frac{r}{c}$ represents the ratio of the cosine to radius. The difference between the respective weights of C and B, in this case, is equal to $\frac{3}{6}$; the ratio of the cosine to radius is equal to—say $\frac{7}{10}$; and the ratio of the weight of A to the weight of C is equal to $\frac{2}{3}$. Hence, if we make R c' equal to three sixths of radius, minus seven tenths of two thirds of three sixths of radius, the line R c' will represent the velocity of C, in opposition to its own given direction, during one second after collision. According to the same Law, C will have a velocity in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of C: which, in this case, is equal to $\frac{4}{9}$. Hence, if we make R c'' equal to four ninths of the cosine R s, the line R c'' will represent the velocity of C, in the given direction of A, during one second after collision. From the lines R c' and R c'', complete the parallelogram, and draw the diagonal R c. Then the point c will be the position of C,

at the end of one second after collision: having described, in that time, the diagonal R c.

To find the position of B, at the end of one second after collision, we must look to Law 3, which says that B will have a velocity in the given direction of A, which,—as compared with the versed sine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of B: which, in this case, is equal to $\frac{4}{9}$. Hence, if we make R b' equal to four ninths of the versed sine E b, the line R b' will represent the velocity of B, in the given direction of A. According to the same Law, B will have a velocity in the given direction of C, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of C to the weight of B, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which $\frac{r}{c}$ represents the ratio of the cosine to radius. The square of the ratio of the weight of C to the weight of B, in this case, is equal to $\frac{9}{36}$; the ratio of the cosine to radius is equal to $\frac{7}{10}$; and the ratio of the weight of A to the weight of C is equal to $\frac{2}{3}$. Hence, if we make R b'' equal to nine thirty-sixths of radius, minus seven tenths of two thirds of nine thirty-sixths of radius, the line R b'' will represent the velocity of B in the given direction of C. From the two lines R b' and R b'', complete the parallelogram, and draw the diagonal R b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal Rb.

Now, R a', plus R a'', multiplied by the weight of A, added to R c', plus R c'', multiplied by the weight of C, added to R b', plus R b'', multiplied by the weight of B; will be equal to A R multiplied by the weight of A, added to C R multiplied by the weight of C: that is, the joint momentum of A, B, and C, after collision, will

be equal to the joint momentum of A and C before collision.

If, in the foregoing example, we suppose radius to be equal to 10 feet, the velocity of both A and C, before collision, would be equal to 10 feet per second. As A is supposed to weigh 2 pounds, the momentum of A, before collision, would be equal to 20; and as C is supposed to weigh 3 pounds, the momentum of C, before collision, would be equal to 30. Hence the joint momentum of A and C before collision would be equal to 50: which is the amount that is to be accounted for after collision. The velocity of A, in this case, in opposition to its own given direction, would be equal to $4\frac{1}{3}$ feet per second, and the momentum of A would be equal to $8\frac{2}{3}$. The velocity of A, in the given direction of C, would be equal to 7 feet per second, and the momentum of A would be equal to 14. The velocity of C, in opposition to its own given direction, would be equal to $2\frac{2}{3}$ feet per second, and the momentum of C would be equal to 8. The velocity of C, in the given direction of A, would be equal to $3\frac{1}{9}$ feet per second, and the momentum of C would be equal to $9\frac{1}{3}$. The velocity of B, in the given direction of A, would be equal to $\frac{1}{3}$ of 1 foot per second, and the momentum of B would be equal to 2. The velocity of B, in the given direction of C, would be equal to $1\frac{1}{3}$ feet, and the momentum of B would be equal to 8. Now, $8\frac{2}{3} + 14 + 8 + 9\frac{1}{3} + 2 + 8 = 50$.

SECTION XIX.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE NINETEENTH CLASS OF CASES: WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF B: IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF C; IN WHICH A AND C COME IN COLLISION WITH B AT THE SAME INSTANT OF TIME; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE, BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 0 AND 90 DEGREES, INCLUSIVE.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the joint weight of A and C, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the respective weights of B and A, and the difference between the weight of B and the joint weight of A and C.

LAW 2. C will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the joint weight of A and C, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the weight of B and the weight of C, and the difference between the weight of B and the joint weight of A and C.

LAW 3. B will have a velocity, in the given direction of A, and also a velocity in the given direction of C; each of which will be proportional to one half of radius, plus one half of the versed sine of the angle. The line that will be actually described by B, in the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, suppose the weight of A to be 2 pounds; the weight of B one pound; and the weight of C 3 pounds. Suppose, also, that A will describe the radius A R, and C the radius C R (Fig. 2, Plate 5), in one second before collision. Then A R C will be the angle formed by the respective lines of direction of A and C; C *s* will be the sine; R *s* the cosine; and A *s* the versed sine of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the joint weight of A and C, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the respective weights of B and A,

and the difference between the weight of B and the joint weight of A and C. The difference, in this case, between the weight of B and the joint weight of A and C, is equal to $\frac{4}{5}$; and the difference between the two differences above named, will be equal to $\frac{3}{10}$. Hence, if we make R *a* equal to four fifths of radius, minus three tenths of the versed sine A *b*, the line R *a* will represent the velocity of A; and the point *a* will be the position of A, at the end of one second after collision: having described, in that time, the right line R *a*.

In order to find the position of C, at the end of one second after collision, we must look to Law 2, which says that C will have a velocity in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the joint weight of A and C, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between two other differences; namely, the difference between the weight of B and the weight of C, and the difference between the weight of B and the joint weight of A and C. The difference, in this case, between the weight of B and the joint weight of A and C, is equal to $\frac{4}{5}$; and the difference between the other two differences is equal to $\frac{2}{15}$. Hence, if we make R *c* equal to four fifths of radius, minus two fifteenths of the versed sine A *b*, the line R *c* will represent the velocity of C during one second after collision; and the point C, in the radius R *b''*, will be the position of C, at the end of one second after collision: having described, in that time, the right line R *c*.

To find the position of B, at the end of one second after collision, we must look to Law 3, which says that B will have a velocity, in the given direction of A, and also a velocity in the given direction of C, each of which

will be proportional to one half of radius, plus one half of the versed sine of the angle. Hence, if we make $R\,b'$ equal to one half of radius, plus one half of the versed sine $A\,s$, the line $R\,b'$ will represent the velocity of B in the given direction of A. And if we make $R\,b''$ equal to one half of radius, plus one half of the versed sine $A\,s$, the line $R\,b''$ will represent the velocity of B, in the given direction of C. From the lines $R\,b''$ and $R\,b'$, complete the parallelogram, and draw the diagonal $R\,b$. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal $R\,b$.

Now, $R\,a$ multiplied by the weight of A, added to $R\,c$ multiplied by the weight of C, added to $R\,b'$, plus $R\,b''$, multiplied by the weight of B; will be equal to A R multiplied by the weight of A, added to C R multiplied by the weight of C: that is, the joint momentum of A, B, and C, after collision, will be equal to the joint momentum of A and C before collision.

If, in the above example, we suppose radius to be equal to 10 feet, the velocity of A, after collision, would be equal to $7\frac{1}{10}$ feet, and its momentum would be equal to $14\frac{1}{5}$. The velocity of C would be equal to $7\frac{2}{5}$ feet, and its momentum would be equal to $22\frac{1}{5}$. The velocity of B, in the given direction of C, would be equal to $6\frac{1}{2}$ feet per second, and its momentum would be equal to $6\frac{1}{2}$. The velocity of B, in the given direction of A, would be equal to $6\frac{1}{2}$ feet per second, and its momentum would be equal to $6\frac{1}{2}$; that is, if the angle be such that the versed sine would be equal to three tenths of radius.

In some cases belonging to this class, A and C, after collision with B, would also come into collision with each other. In such cases the directions and velocities of A and C, after collision, would be different from those

above assigned to them; but their joint momentum would not be altered.

SECTION XX.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE TWENTIETH CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF A IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF B; IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF C; IN WHICH A AND C COME IN COLLISION WITH B AT THE SAME INSTANT OF TIME; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES, INCLUSIVE.

In every case belonging to this class, the results of collision will be determined by the following Laws:

Law 1. A will have a velocity, in its own given direction, which,—as compared with the versed sine of the angle,—will be proportional to the difference between the respective weights of B and A. A will also have a velocity, in opposition to its own given direction, which,—as compared with the cosine of the angle,—will be propor-

tional to the difference between the respective weights of A and C. A will also have a velocity, in the given direction of C, that will be proportional to the cosine of the angle. The line that will be actually described by A, in the given time after collision, will coincide with the diagonal of a parallelogram completed from the line which represents the velocity of A in the given direction of C, and that which represents the resultant of the two opposite velocities aforesaid.

LAW 2. C will have a velocity, in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the weight of C, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which $\frac{r}{c}$ represents the ratio of the cosine to radius. C will also have a velocity, in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of C. The line that will be actually described by C, in the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

LAW 3. B will have a velocity, in the given direction of A, that will be proportional to the versed sine of the angle. B will also have a velocity, in the given direction of C, that will be proportional to radius, minus a quantity,—as compared with the cosine of the angle,—proportional to the ratio of the weight of A to the weight of C. The line that will be actually described by B, in the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

For example, suppose that A weighs 3 pounds, B 1 pound, and C 4 pounds. Suppose, also, that A will

describe the radius A R, and C the radius C R (Figs. 1, and 2, Plate 6), in one second before collision. Then A R C will be the angle formed by the respective lines of direction of A and C; C s will be the sine; R s the cosine; and E s the versed sine, of that angle.

Now, in order to find the position of A, at the end of one second after collision, we must look to Law 1, which says that A will have a velocity, in its own given direction, which,—as compared with the versed sine of the angle,—will be proportional to the difference between the respective weights of B and A. That difference, in this case, is equal to $\frac{2}{3}$. Hence, if we make R a' equal to two thirds of the versed sine E s, the line R a' will represent the velocity of A, in its own given direction, during one second after collision. The same Law says that A will have a velocity, in opposition to its own given direction, which,—as compared with the cosine of the angle,—will be proportional to the difference between the respective weights of A and C. In this case, that difference is equal to $\frac{1}{4}$. Hence, if we make $a'\,a''$ equal to one fourth of the cosine R s, the line $a'\,a''$ will represent the velocity of A in opposition to its own given direction; and the line R a'' will represent the resultant of the velocities in the opposite directions aforesaid. The same Law says that A will have a velocity in the given direction of C, that will be proportional to the cosine of the angle. Hence, if we make R a''' equal to the cosine R s, the line R a''' will represent the velocity of A in the given direction of C. From the lines R a'' and R a''', complete the parallelogram, and draw the diagonal R a. Then the point a will be the position of A, at the end of one second after collision: having described, in that time, the diagonal R a.

In order to find the position of C, at the end of one

second after collision, we must look to Law 2, which says that C will have a velocity, in its own given direction, which,—as compared with radius,—will be proportional to the difference between the weight of B and the weight of C, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which $\frac{r}{c}$ represents the ratio of the cosine to radius. The difference, in this case, between the respective weights of B and C is equal to $\frac{3}{4}$; the ratio of the cosine to radius, in Fig. 2, $\frac{7}{10}$; and the ratio of the weight of A to the weight of C is equal to $\frac{3}{4}$. Hence, if we make R c'' equal to three fourths of radius, minus seven tenths of three fourths of the cosine R s, the line R c will represent the velocity of C, in its own given direction, during one second after collision. According to the same Law, C will have a velocity, in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of C. That square, in this case is equal to $\frac{9}{16}$. Hence, if we make R c equal to nine sixteenths of the cosine R s, the line R c' will represent the velocity of C, in the given direction of A. From the lines R c' and R c'', complete the parallelogram, and draw the diagonal R c. Then the point c will be the position of C, at the end of one second after collision: having described, in that time, the diagonal R c.

In order to find the position of B, at the end of one second after collision, we must look to Law 3, which says that B will have a velocity, in the given direction of A, that will be proportional to the versed sine of the angle. Hence, if we make R b' equal to the versed sine E s, the line E s will represent the velocity of B, in the given direction of A, during one second after collision. According to the same Law, B will have a velocity, in

the given direction of C, that will be proportional to radius, minus a quantity,—as compared with the cosine of the angle,—proportional to the ratio of the weight of A to the weight of C. That ratio, in this case, is equal to $\frac{3}{4}$. Hence, if we make R b'' equal to radius, minus three fourths of the cosine R b, the line R b'' will represent the velocity of B, in the given direction of C. From the lines R b' and R b'', complete the parallelogram, and draw the diagonal R b. Then the point b will be the position of B, at the end of one second after collision: having described, in that time, the diagonal R b.

Now, R a', plus $a' a''$, plus R a''', multiplied by the weight of A, added to R c', plus R c'', multiplied by the weight of C, added to R b, plus R b'', multiplied by the weight of B; will be equal to A R multiplied by the weight of A, added to C R multiplied by the weight of C: that is, the joint momentum of A, B, and C, after collision, will be equal to the joint momentum of A and C before collision.

If, in Fig. 2 of the above example, we suppose radius to be equal to 10 feet, the velocity of A, after collision, in its own given direction, would be equal to 2 feet, and its momentum would be equal to 6. The velocity of A, in opposition to its own given direction, would be equal to $1\frac{3}{4}$ feet, and its momentum would be equal to $5\frac{1}{4}$. The velocity of A, in the given direction of C, would be equal to 7 feet (the supposed value of the cosine), and its momentum would be equal to 21. The velocity of C, in its own given direction, would be equal to $3\frac{9}{16}$ feet, and its momentum would be equal to $14\frac{1}{4}$. The velocity of C, in the given direction of A, would be equal to $3\frac{15}{16}$ feet, and its momentum would be equal to $15\frac{3}{4}$. The velocity of B, in the given direction of A, would be equal to 3 feet, and its momentum would be equal to 3. The ve-

locity of B, in the given direction of C, would be equal to $4\frac{3}{4}$ feet, and its momentum would be equal to $4\frac{3}{4}$.

SECTION XXI.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE TWENTY-FIRST CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF C IS GREATER THAN, OR EQUAL TO, THE WEIGHT OF A; IN WHICH THE WEIGHT OF B IS GREATER THAN THE WEIGHT OF EITHER C OR A, BUT LESS THAN THE JOINT WEIGHT OF A AND C; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF C; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND IN WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 0 AND 90 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will have a velocity in its own given direction, which,—as compared with the cosine of the angle,—will be proportional to the difference between the joint weight of A and C and the weight of B. A will also have a velocity in opposition to its own given direction, which,—as compared with the versed sine of the angle,—will be proportional to the difference between the respective weights of A and B. The line that will be actually described by

A, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

LAW 2. C will have a velocity in its own given direction, which,—as compared with the cosine of the angle,—will be proportional to the difference between the weight of B and the joint weight of A and C. C will also have a velocity in opposition to its own given direction, which,—as compared with the versed sine of the angle,—will be proportional to the difference between the respective weights of C and B. The line that will be actually described by C, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

LAW 3. B will have a velocity, in the given direction of A, that will be proportional to $\frac{1}{2}$ of radius, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between $\frac{s}{r}$ of radius and $\frac{1}{2}$ of radius: in which $\frac{s}{r}$ represents the square of the ratio of the weight of A to the weight of B. This quantity will be plus, instead of minus, when $\frac{s}{r}$ is greater than $\frac{1}{2}$. B will also have a velocity, in the given direction of C, that will be proportional to $\frac{1}{2}$ of radius, minus a quantity,—as compared with the versed sine of the angle,—proportional to the difference between $\frac{s}{r}$ of radius and $\frac{1}{2}$ of radius: in which $\frac{s}{r}$ represents the square of the ratio of the weight of C to the weight of B. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

SECTION XXII.

DEVOTED TO THE ENUNCIATION OF THE LAWS WHICH DETERMINE THE RESULTS OF COLLISION IN THE TWENTY-SECOND CLASS OF CASES; WHICH CONSISTS OF THOSE IN WHICH THE WEIGHT OF C IS EQUAL TO, OR GREATER THAN, THE WEIGHT OF A; IN WHICH THE WEIGHT OF B IS GREATER THAN THE WEIGHT OF EITHER A OR C, BUT LESS THAN THE JOINT WEIGHT OF A AND C; IN WHICH THE VELOCITY OF A IS EQUAL TO THE VELOCITY OF C; IN WHICH B IS AT REST AT THE TIME OF COLLISION; IN WHICH THE POSITION OF B, AT THE TIME OF COLLISION, IS SUCH, THAT THE RESPECTIVE LINES OF DIRECTION IN WHICH A AND C MOVE BEFORE COLLISION, WILL PASS THROUGH THE CENTER OF GRAVITY OF B; AND I WHICH THE SAID LINES OF DIRECTION MAKE WITH EACH OTHER ANY ANGLE WHATEVER BETWEEN 90 AND 180 DEGREES, INCLUSIVE.

IN every case belonging to this class, the results of collision will be determined by the following Laws:

LAW 1. A will have a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of A and B, minus a quantity,—as compared with the cosine of the angle,—proportional to the difference between two other differences; namely, the difference between the respective weights of A and B, and the difference between the respective weights of A and C. A will also have a velocity, in the given direction of C, that will be proportional to the cosine of the angle. The line

that will be actually described by A, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

LAW 2. C will have a velocity in opposition to its own given direction, which,—as compared with radius,—will be proportional to the difference between the respective weights of C and B, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which equation $\frac{r}{c}$ represents the ratio of the cosine to radius. C will also have a velocity in the given direction of A, which,—as compared with the cosine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of C. The line that will be actually described by C, during any given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid.

LAW 3. B will have a velocity in the given direction of A, which,—as compared with the versed sine of the angle,—will be proportional to the square of the ratio of the weight of A to the weight of B. B will also have a velocity in the given direction of C, which,—as compared with radius,—will be proportional to the square of the ratio of the weight of C to the weight of B, minus $\frac{r}{c}$ of a quantity proportional to the ratio of the weight of A to the weight of C: in which equation $\frac{r}{c}$ represents the ratio of the cosine to radius. The line that will be actually described by B, during the given time after collision, will coincide with the diagonal of a parallelogram completed from the two lines which represent the directions and velocities aforesaid

SECTION XXIII.

DEVOTED TO THE CONSIDERATION OF WHAT IS KNOWN AS THE FIRST LAW OF MOTION, AS DRAWN UP BY SIR ISAAC NEWTON IN HIS "MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY"; AND WHICH READS AS FOLLOWS:

"LAW 1.–*Every body perseveres, in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*"

NEWTON, before enunciating this Law, gives eight definitions of words, in order, he says, to explain the sense in which he would have them understood. The third of the said eight definitions is as follows:

"*The* Vis Infita, *or Innate Force of Matter, is a power of resisting, by which every body, as much as in it lies, endeavors to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*"

"This force is ever proportional to the body whose force it is; and differs nothing from the inactivity of the mass, but in our manner of conceiving it. A body from the inactivity of matter, is not without difficulty put out of its state of rest or motion. Upon which account this *Vis Infita*, may, by a most significant name, be called *Vis Inertia*, or Force of Inactivity. But a body exerts this force only, when another force impressed upon it, endeavors to change its condition; and the exercise of this force may be considered both as resistance and im-

pulse. It is resistance in so far as the body, for maintaining its present state withstands the force impressed; it is impulse, in so far as the body, by not easily giving way to the impressed force of another, endeavors to change the state of that other."

This definition explains what Newton means by saying that every body perseveres in its state of rest, or of uniform motion in a right line. He means: 1. That when a body is in a state of rest, it will continue in that state until some force puts it in motion. 2. That when a body is moving in a right line with a uniform velocity, it will continue to move in a right line with a uniform velocity, until some force compels it to change its velocity and direction. 3. That when a body is in a state of rest, or when it is moving in a right line with a uniform velocity, it possesses a property by virtue of which it is enabled to resist, to some extent, any force which endeavors to change its condition. 4. That the resisting force of a body, or the power which it is capable of exerting in opposition to any force that endeavors to change its condition, is always proportional to the weight of the body.

Now, the first, second, and third, of the four propositions above set forth, are not affected by the Laws enunciated in the preceding sections of this work; but those Laws show that the fourth proposition is not true: that is, they show that the resisting force of a body is not proportional to the weight of the body.

As the weight of a body, at the surface of the earth, is a definite quantity, the resisting force of that body must be a definite quantity also, if it be proportional to the weight. And if the resisting force be a definite quantity, it must be the same in amount, for the same body, in all cases. Now, suppose a case in which the weight of A is equal to the weight of B, and in which B

is at rest at the time of collision. Suppose also that the respective weights of A and B are each equal to 1 pound, and that the velocity of A before collision is equal to 10 feet per second. In this case, according to the Laws of sections 1 and 2, B, at the time of collision, will take off the whole of A's momentum,—which is equal to 10,—and A will be brought to a state of rest. Hence the resisting force of B, in this case, is equal to 10. Now, suppose the velocity of A before collision to be equal to 100 feet per second, instead of 10 feet per second; and then the momentum of A before collision will be equal to 100 instead of 10. According to the same Laws, B, in this case also, will take off the whole of A's momentum,—which is equal to 100,—and A will be brought to a state of rest. Hence the resisting force of B, in this case, will be equal to 100; which is ten times greater than it was in the former case. Furthermore, if by increasing the velocity of A before collision, the momentum of A be made a thousand, a million, or any other number of times greater than ten, B, at the time of collision, will take off the whole of that momentum, and A will be brought to a state of rest. Hence we perceive that when the weight of A is equal to the weight of B, and B is at rest at the time of collision, the resisting force of B is not at all proportional to its own weight, nor even a definite quantity; but proportional to the velocity of A; and therefore a variable quantity.

By the same Laws, when the weight of A is greater than the weight of B, B will not, in any case, take off the whole of A's momentum, but a quantity proportional to the ratio of the weight of B to the weight of A. But in a series of cases in which the respective weights of A and B remain constant, while the velocity of A is made to undergo every possible degree of variation, the resist-

ing force of B will, as in the former case, be proportional to the velocity of A; and therefore a variable quantity.

By the same Laws, when the weight of A is less than the weight of B, B will take off the whole of A's momentum in the given direction of A, and, besides this, it will cause A to be reflected back in the opposite direction, with a momentum proportional to the difference between the respective weights of A and B. Hence the resisting force of B, in any given case, will be equal to the momentum of A before collision,—whatever that momentum may be,—plus a quantity,—as compared with the momentum of A before collision,—proportional to the difference between the respective weights of A and B. But in a series of cases in which the respective weights of A and B remain constant, while the velocity of A is made to undergo every possible degree of variation, the resisting force of B will be proportional to the velocity of A; and therefore a variable quantity.

Hence it is clear that the resisting force of a body is not, as Newton supposes, proportional to the body whose force it is, nor even a definite quantity, but subject to every possible degree of variation.

SECTION XXIV.

DEVOTED TO THE CONSIDERATION OF WHAT IS KNOWN AS THE SECOND LAW OF MOTION, AS DRAWN UP BY SIR ISAAC NEWTON, AND WHICH READS AS FOLLOWS:

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.* If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subducted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both."

THE most important proposition contained in the foregoing Law, is that which asserts that the motions impressed by different forces are always proportional to the forces by which those motions are impressed. And it can be shown that the proportion holds good in only a few particular cases.

When the force consists of a ponderable body, moving with any velocity whatever, there are three methods by which that force may be increased. 1. It may be in-

creased by increasing the velocity of the body. 2. It may be increased by increasing the weight of the body. 3. It may be increased by increasing both the weight and velocity of the body. Hence, the force will consist of a combination of weight and velocity; and any given force may be doubled by doubling the weight; or, it may be doubled by doubling the velocity; or, it may be doubled by adding either equal or unequal portions of weight and velocity. And hence the question arises, Will a given force which has been doubled by doubling its weight, have the same effect to move a given body, that it would have had if the force had been doubled by doubling its velocity? The answer to this question is contained in the Laws of the first two sections of this work. Suppose a case in which the weight of A is equal to 1 pound, and the velocity of A before collision equal to 10 feet per second; in which the weight of B is equal to 4 pounds, and in which B is at rest at the time of collision. This case comes under the Laws of Section 1. Law 2, of that section, says that the velocity of B,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B: which, in this case, is equal to $\frac{1}{16}$. Hence the velocity of B would be equal to one sixteenth of 10 feet per second. Now, if we suppose the force to be doubled by making the weight of A equal to 2 pounds, the velocity that A would impress upon B would,—according to the same Law,—be equal to four sixteenths of 10 feet per second. And if the weight of A were made equal to 3 pounds, the velocity that A would impress upon B would be equal to nine sixteenths of 10 feet per second. And if the weight of A were made equal to 4 pounds, the velocity that A would impress upon B, would be equal to sixteen sixteenths of 10 feet

per second: that is the velocity of B would be equal to the given velocity of A when the weight of A became equal to the weight of B. Hence we perceive that the velocities which would be impressed upon B by A, in these cases, would not be proportional to the forces, but proportional to the squares of the forces.

We have seen that when the weight of A is equal to 1 pound, and the velocity of A, before collision, equal to 10 feet per second, A would impress upon B a velocity equal to one sixteenth of 10 feet per second. If, now, we make no alteration in the weight of A, but make the force twice as great as before, by making the velocity of A, before collision, equal to 20 feet per second, instead of 10; the velocity that will be impressed upon B by A, will, according to the same Law, be equal to one sixteenth of 20 feet per second. And when the force is made three times as great, by making the velocity of A equal to 30 feet per second, the velocity that will be impressed upon B by A, will be equal to one sixteenth of 30 feet per second: and so on. Hence we perceive that when the force is increased by increasing,—not the weight,—but the velocity only, the velocities that will be impressed upon B by A, will be directly proportional to the forces.

In all similar cases,—that is when the weight of A is less than, or equal to, the weight of B, and B is at rest at the time of collision; if the force be increased by increasing both the weight and velocity of A, the velocities that will be impressed upon B by A, will never be proportional either directly to the forces, or to the squares of the forces. They will always be greater than the direct proportion of the forces, and always less than the proportion of the squares of the forces. How much greater than the one, or how much less than the other, will de-

pend upon the relative proportions of the weight and velocity that is added to the force. But whatever may be the weight, or whatever may be the velocity that is either added to, or taken from, the force; the velocity which that force will impress upon B, may always be found by the Laws of Sections 1 and 2.

Now suppose a case in which the weight of A is equal to 2 pounds, and the velocity of A before collision is equal to 10 feet per second; in which the weight of B is equal to 1 pound, and in which B is at rest at the time of collision. This case comes under the Laws of Section 2. Law 2, of that section, says that the velocity which will be impressed upon B by A, in this case, will be exactly equal to the velocity which A had before collision; namely, 10 feet per second. If, now, the velocity of A remain unaltered, and the force be made ten times greater than before, by making the weight of A equal to 20 pounds; or, if the force be made one hundred times greater than before by making the weight of A equal to 200 pounds; or, if the force be increased to any extent whatever by adding to the weight of A; the velocity that will be impressed upon B by A, will,—according to the same Law,—in each case be equal to the velocity which A had before collision; namely, 10 feet per second. Hence we perceive that the velocities impressed upon B by A, in these cases, are not at all proportional to any power of the force, but proportional to the velocity of A. We also perceive that equal velocities may be impressed, upon the same body, by two forces of which one may be a thousand or a million times greater than the other.

On the other hand, if, instead of increasing the force by increasing the weight of A, we increase the force by increasing the velocity of A before collision; the ve-

locities that will be impressed upon B by A, will be directly proportional to the forces; for the velocity of B after collision will always be exactly equal to the velocity of A before collision.

Now suppose a case in which the weight of A is equal to 20 pounds, and the velocity of A, before collision, equal to 10 feet per second; in which the weight of B is also equal to 20 pounds, and in which B is at rest at the time of collision. According to the Laws of Sections 1, 2, and 3, the velocity that would be impressed upon B by A, in this case, would be equal to 10 feet per second. If, now, the weight of A be made four times less than before: that is, if the weight of A be made equal to 5 pounds instead of 20 pounds,—the velocity of A, before collision, must be made 16 (the square of four) times greater than before,—that is, it must be made equal to 160 feet per second, instead of 10 feet per second,—in order that A may impress upon B a velocity equal to that which A impressed upon B, when the weight of A was equal to the weight of B; namely, 10 feet per second. For the Law which determines the result in this case is, that the velocity of B,—as compared with the velocity of A before collision,—will be proportional to the square of the ratio of the weight of A to the weight of B. That square, in this case, is equal to $\frac{1}{16}$; and 10 feet per second is equal to $\frac{1}{16}$ of 160 feet per second. Now, a body weighing 20 pounds, and moving with a velocity of 10 feet per second, constitutes a force equal to 200; and a body weighing 5 pounds, and moving with a velocity of 160 feet per second, constitutes a force equal to 800. And yet the former will impress upon B a velocity exactly equal to that which the latter would impress upon B.

Furthermore, if the weight of A be made twenty times

less than before: that is, if the weight of A be made equal to 1 pound, instead of 20 pounds,—the velocity of A before collision must be made four hundred (the square of 20) times greater than before: that is, it must be made equal to 4,000 feet per second, instead of 10 feet per second, in order that A may impress upon B the same velocity that it did when A weighed 20 pounds, and moved with a velocity of 10 feet per second. Now one of these forces is equal to 200, and the other is equal to 4,000, and yet one would impress upon B precisely the same velocity as the other.

Thousands of other cases might be pointed out, each of which would show that the motions impressed upon bodies are not proportional to the forces by which those motions are impressed; but those already pointed out are quite sufficient. Indeed, in order to overthrow Newton's Second Law of Motion, it was not necessary to point out a single case in which that Law did not hold good; for that Law is absurd upon its face, inasmuch as it implies that one body, by direct action, can impress upon another body a velocity greater than its own.

The true Law in regard to the proportion between the force and the motion impressed is, that the motion impressed upon bodies, as compared with the forces by which those motions are impressed, is subject to every possible degree of variation between, and including, two extremes; in one of which the motion is proportional to the force; and in the other infinitely disproportional to the force.

SECTION XXV.

DEVOTED TO THE CONSIDERATION OF NEWTON'S THIRD LAW OF MOTION, WHICH READS AS FOLLOWS:

"*To every Action there is always opposed an equal Reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*"

THE amount of the action of A upon B, and the amount of the reaction of B upon A, in any given case, may be determined exactly by the Laws enunciated in the preceding sections. By consulting those Laws, it will be seen that when the weight of A is equal to, or greater than, the weight of B, the action of A upon B, and the reaction of B upon A, will, in every case, be equal to each other. For example, suppose a case in which the weight of A is equal to the weight of B; in which the velocity of A, before collision, is equal to 10 feet per second; and in which B is at rest at the time of collision. Now, according to the Laws of Sections 1, 2, and 3, A, in this case, will impress upon B a velocity equal to its own; namely, 10 feet per second. And as the weights are equal, the momentum of B, after collision, will be equal to the momentum of A before collision. According to the same Law, A, after collision, will have no motion at all. Hence, in this case a mutual exchange of conditions takes place between the bodies: that is, B, which was at rest, is made to move, by A, with a velocity of 10 feet per second: and A, which moved with a

velocity of 10 feet per second, is brought by B to a state of rest. And hence the action and reaction are equal.

The proof of the equality of the action and reaction, in the foregoing example, does not consist alone of the fact that the momentum gained by B is equal to the momentum lost by A ; but it consists of that fact, together with the additional fact, that the momentum gained by B has the same direction as the momentum which is lost by A. And as the same is true with respect to every case in which the weight of A is equal to, or greater than, the weight of B, it follows that the action and reaction are equal in all those cases.

The fact that the momentum gained by B is equal to the momentum lost by A, is not, of itself, sufficient to prove the equality of the action and reaction. It is also necessary to show that the momentum gained by B is equal to the whole of the momentum lost by A in the direction in which B moves. It can be shown that such is the case when the weight of A is equal to, or greater than, the weight of B; but such is not the case when the weight of A is less than the weight of B. The momentum gained by B, in these cases, is equal to the momentum lost by A, but it does not from thence follow that the action and reaction are equal; for it can be shown that the momentum gained by B, is not equal to the whole of the momentum lost by A, in the direction in which B moves. And this fact is, of itself, sufficient to show that in every case in which the weight of A is less than the weight of B, the reaction of B upon A is always greater than the action of A upon B.

When the weight of A is greater than the weight of B, A imparts to B, at the time of collision, a portion of its momentum, and follows on after B with the remainder. The direction of all the motion after collision is the same

as it is before collision. And as the momentum gained by B is equal to the momentum lost by A, and in the same direction, the equality of the action and reaction is evident. But when the weight of A is less than the weight of B, A imparts to B, at the time of collision, a portion of its momentum,—as in the former case; but A does not then, as in the former case, follow on after B with the remainder. But A loses the whole of its momentum in its given direction, and is then reflected back, from the point of collision, in the opposite direction. Hence there is a momentum lost, and a momentum gained, by A. The momentum lost by A consists of the whole of the momentum which A had in its given direction before collision. The momentum gained by A consists of the whole of the momentum which A has, after collision, in the opposite direction. Now the excess of the reaction of B upon A, over the action of A upon B, in any given case, will always be equal to twice the sum of the momentum gained by A in that case. For example, suppose a case in which the weight of A is equal to 1 pound and the weight of B equal to 3 pounds; in which the velocity of A before collision is equal to 10 feet per second, and in which B is at rest at the time of collision. The momentum of A, before collision, in this case, will be equal to 10, and the momentum of B, after collision, will be proportional to the ratio of the weight of A to the weight of B. Hence, the momentum of B, after collision, will be equal to one third of 10: that is, the amount of the action of A upon B will be equal to $3\frac{1}{3}$. A will be reflected back from the point of collision, and the momentum of A will be proportional to the difference between the respective weights of A and B. Hence the momentum of A will be equal to two thirds of 10: that is, the reaction of B upon A, in one direction alone,

will be equal to $6\frac{2}{3}$: which is twice as great as the action of A upon B. But this is only one half of the sum of the reaction of B upon A; for A had a momentum, before collision, in its given direction, equal to 10. And as A imparted to B only one third of 10, it follows that A lost a momentum, in its given direction, equal to two thirds of 10: equal to $6\frac{2}{3}$. Hence we perceive that the reaction of B upon A, in this case, is four times as great as the action of A upon B. And hence, Newton's Third Law of Motion is only half true: that is, it holds good in every case in which the weight of A is equal to, or greater than, the weight of B; but it does not hold good in a single case in which the weight of A is less than the weight of B.

SECTION XXVI.

DEVOTED TO THE CONSIDERATION OF NEWTON'S FIRST COROLLARY TO HIS THREE LAWS OF MOTION; WHICH READS AS FOLLOWS:

"A body by two forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides, by those forces acting apart."

THIS corollary was made the foundation of what is now known as the Theory of the Composition of Forces; which, as drawn up by Laplace, is as follows:

"The effect of a force acting on a material point, is, if no obstacle intervenes, to put it in motion. The direction of the force is the right line which it tends to make the

point describe. It is evident that if two forces act in the same direction, their effect to move the point is the sum of the forces, and that when they act in opposite directions, the point is moved by a force represented by their difference, so that if the forces were equal, the point would remain at rest.

"If the directions of the two forces make an angle with each other, a force results, the direction of which is intermediate between the directions of the composing forces, and it can be demonstrated by geometry alone, that if from the point of concourse of these forces, and in their respective directions, right lines be assumed which represent them, and if then the parallelogram of which these lines are adjacent sides, be completed, its diagonal will represent their resultant, both in quantity and direction."

As the whole of Newton's first corollary is included in the Theory of the Composition of Forces, and as the principal object here is to show how near that theory is to being wholly false, it will not be necessary to consider Newton's first corollary separately.

The Theory of the Composition of Forces,—which is the joint production of Newton and Laplace,—supposes that when two forces act upon a body at the same time, and in the same direction, their effect to move the body will be equal to the sum of the forces: that is, it supposes that if one given force would impress upon a given body a velocity equal to 5 feet per second; and that if another given force would impress upon the same body a velocity equal to 10 feet per second; then, when both forces acted upon the body at the same time, and in the same direction, they would impress upon the body a velocity equal to 15 feet per second. This proposition is derived from Newton's Second Law of Motion, and it is

only saying in other words that the motions impressed upon the same body, by different forces, are always proportional to the forces; whether they act upon it separately or together. Now the Laws which determine the results of collision, in every case comprehended by the Theory of the Composition of Forces, and in which the force consists of one or more ponderable bodies, are given in the preceding sections of this work. Those Laws declare that there is but one class of cases in which the proposition holds good; and that is when the respective weights of A, B, and C, are proportional to the sides of a right-angled triangle: that is, when the weight of B is proportional to the side which subtends the right angle, and the weights of A and C proportional to the sides which contain the right angle. When the weights are in these proportions the velocity that will be impressed upon B, by A and C, when they both act upon it at the same time, will be exactly equal to the sum of the velocities that would be impressed upon B by their separate action: Provided that the respective velocities of A and C were equal to each other when they acted separately; and provided, further, that they have the same velocity when acting together as when acting separately. For, in cases of this kind, the respective weights of A and C will each be less than the weight of B; and their joint weight will be greater than the weight of B. And the Law which determines the result of their separate action upon B, is, that the velocity impressed upon B, in each case,—as compared with the velocity of A, or C, before collision,—will be proportional to the square of the ratio of the weight of A, or the weight of C, to the weight of B. Now the sides of a right-angled triangle may be proportional to the numbers 3, 4, and 5. And if, in a given case, the weights of

A, C, and B, respectively, were proportional to those numbers, then the ratio of the weight of A to the weight of B would be $\frac{3}{5}$; and the ratio of the weight of C to the weight of B would be $\frac{4}{5}$. Hence, when A acted alone upon B,—say with a velocity of 10 feet per second,—the velocity that would be impressed upon B by A, would be equal to $\frac{9}{25}$ of 10 feet per second; and when C acted alone upon B, the velocity that would be impressed upon B by C, would be equal to $\frac{16}{25}$ of 10 feet per second. And these sums added together are equal to $\frac{25}{25}$ of 10 feet per second. If, now, the weight of C be added to the weight of A, their joint weight will be greater than the weight of B. And the Law which determines the results of collision when the weight of A is greater than the weight of B, is, that the velocity of B, after collision, will be equal to the velocity which A had before collision: which, in this case, is equal to 10 feet per second. And therefore in this, and in all similar cases,—that is, when the respective weights of A, B, and C, are such that the square of the weight of A, added to the square of the weight of C, is equal to the square of the weight of B, and B is at rest at the time of collision,—the velocity that will be impressed upon B by the joint action of A and C, will be equal to the sum of the velocities that would be impressed upon B by their separate action.

Now suppose the weights and velocities of A and C to remain the same, and that the weight of B is made equal to 6 pounds, instead of 5. In this case the velocity that would be impressed upon B by A, when acting alone, would be equal to $\frac{9}{36}$ of 10 feet per second; and the velocity that would be impressed upon B by C, when acting alone, would be equal to $\frac{16}{36}$ of 10 feet per second. And the velocity that would be impressed upon B, by A

and C, when they both acted upon it at the same time, would be equal to $\frac{56}{36}$ of 10 feet per second: which is $\frac{11}{36}$ of 10 feet greater than the sum of the velocities impressed by A and C, when acting separately. If the weight of B were made equal to 7 pounds, instead of 6, the difference would be equal to $\frac{24}{49}$ of 10 feet per second. And if the weight of B were made equal to 8 pounds, instead of 7, the difference would be equal to $\frac{24}{64}$ of 10 feet per second; and so on. There will be a difference in every case whatever, except in the example first above given: that is, when the square of the weight of A, added to the square of the weight of C, is equal to the square of the weight of B, the velocity that will be impressed upon B, by A and C, when they both act upon B at the same time, will be equal to the sum of the velocities that would be impressed by their separate action, and in every other case it will be unequal. Hence that part of the Theory of the Composition of Forces which supposes that the velocity impressed upon a body by two forces which act upon it at the same time, will be equal to the sum of the velocities that would be impressed upon it by those forces when acting separately, is,—when the forces consist of ponderable matter,—true with respect to one particular class of cases, and false with respect to all others whatsoever.

The Theory of the Composition of Forces supposes, further, that when two equal forces act upon a body at the same time, in opposite directions, the body will remain at rest: so far as the action of those forces are concerned. This proposition holds good with respect to one particular class of cases only.

Suppose a case in which the weight of B is equal to 10 pounds, and in which B is at rest at the time of collision; in which the respective weights of A and

C are each equal to 10 pounds, and in which the respective velocities of A and C, before collision, are each equal to 10 feet per second. Now, according to the Laws enunciated in the preceding sections, A and C, when acting separately, will impress equal velocities upon B, and when acting at the same time, in opposite directions, B will remain at rest. And so it will in every case in which the respective weights of A and C are equal to each other, and in which the respective velocities of A and C are also equal to each other.

Now suppose the weight of A to be reduced from 10 pounds to 1 pound; and suppose the velocity of A to be increased from 10 feet per second to 100 feet per second. The forces will still be equal to each other; but in consequence of those forces being composed of different proportions of weight and velocity, they will now impress unequal velocities upon B. C will impress upon B a velocity equal to its own; namely, 10 feet per second; and A will impress upon B a velocity equal to 1 foot per second. Hence when A and C act upon B at the same time, in opposite directions, B, instead of remaining at rest, will move, in the given direction of C, with a velocity equal to 9 feet per second. And so in every case in which the forces are equal to each other, but in which those forces are compounded of unequal quantities of weight and velocity, B, instead of remaining at rest, will move in the direction of the force which is composed of the greater weight and the lesser velocity. Hence, that part of the Theory of the Composition of Forces which supposes that a body will remain at rest, when acted upon at the same time by two equal forces, in opposite directions, is true with respect to one particular class of cases, but is false with respect to all others whatsoever.

Furthermore, the Theory of the Composition of Forces supposes that a body acted upon by two forces at the same time, will describe the diagonal of a parallelogram, in the same time that it would describe the sides by those forces acting apart.

By the Laws enunciated in the preceding sections, it may be determined what the velocity and direction of a body will be,—in any given case whatsoever,—that is acted upon by two forces at the same time, or by either force alone, whatever may be the respective weights and velocities of the bodies, or whatever may be the angle formed by the lines of direction in which the bodies move. An examination of those Laws will show that the proposition set forth at the commencement of this paragraph, holds good with respect to a case in which the respective weights of A, B, and C, are such that the square of the weight of A, added to the square of the weight of C, is equal to the square of the weight of B; and in which the angle formed by the lines of direction of A and C, is equal to zero. And those Laws will show that it also holds good when the angle is a right angle, whatever may be the respective weights of A, B, and C. At all other angles the time of describing the diagonal by both forces conjoined (or rather the time of describing a space equal to the diagonal, for the body will not,—except in a few cases,—move in the direction of the diagonal of a parallelogram completed from the lines which the body would describe, in the given time, by those forces acting apart), as compared with the time of describing the sides of those forces acting apart, will vary between two extremes, in one of which it will be only half, and in the other twice as great. All the variations between the two extremes may be produced at a given angle: namely, when the angle is the least possible, by

varying the proportion of the weights of the bodies. Hence those variations are not due to the variations of the angle alone, but also to the variation of the weights of the bodies. The time of describing the diagonal will vary with every variation of the angle, and also with every variation of the proportion of the weights of the bodies, and can not, therefore, be expressed by a general law; but the time of describing the diagonal in any given case, may be found by consulting the Laws enunciated in the preceding sections of this work.

Those Laws and the Theory of the Composition of Forces agree, when the angle formed by the lines of direction in which the forces move is a right angle; and the reason of it is, that when the angle is a right angle, the force acting in one of the directions neither coincides with, nor is it at all opposed to, the force acting in the other direction. Hence this force is as free to impress a motion upon the body, as it would have been if the other force had not existed. But when the angle is either greater or less than a right angle, the force acting in one of the directions must, to some extent, either coincide with, or be opposed to, the force acting in the other direction. Hence this force is not as free to impress a motion upon the body, as it would have been if the other force had not existed; for that other force must necessarily interfere with it, and in such a manner, that the motion impressed upon the body, will be either more or less than what it would have been, if the force had been acting upon the body alone.

Thus we perceive that although the Theory of the Composition of Forces is not altogether false, yet it is so near to being wholly false, that its scientific value is of no account whatever.

SECTION XXVII.

DEVOTED TO THE CONSIDERATION OF SIR ISAAC NEWTON'S METHOD OF INVESTIGATING THE LAWS OF MOTION.

1. In his third definition—quoted in the preceding section—Newton asserts that the inertia, or force of resistance of a body, is proportional to the weight of the body. Now, as he neither knew, nor pretended to know, how to determine what the resisting force of a body would be in any given case whatever, his assertion that it is always proportional to the weight of the body, could have had no other foundation than a conjecture. But instead of informing us that it was a conjecture, he asserts, without qualification, that such is the case; and there he leaves the matter.

2. In enunciating his Second Law of Motion, Newton asserts that the motion impressed upon a body is always proportional to the force. Now, if Newton had known how to determine the quantity of motion that would be impressed upon any given body, by any given force whatever, he would have been entitled to say whether the motions impressed were proportional to the forces or not. But he neither knew, nor pretended to know, how to determine the quantity of motion that would be impressed upon a body by a given force, in any case whatever; and therefore his assertion that the motions impressed are always proportional to the forces, could have had no other foundation than a conjecture. If he had told us plainly that he knew little or nothing about the

matter with any degree of certainty, but that it was a conjecture of his, or the conjecture of some one else; that the motions impressed are proportional to the forces, no one would have been led astray; but instead of that, he declares the conjecture to be an axiom, or law of motion; and thereby his followers have been greatly deceived: for the conjecture is not more than half true.

3. In his Third Law of Motion Newton asserts that action and reaction are always equal to each other. Now, if he had known how to determine the amount of the action of A upon B, and also the amount of the reaction of B upon A in all cases, he would have been entitled to say whether they were equal or unequal; but he neither knew, nor pretended to know, how to determine the amount of either the one or the other in a single case. And therefore his assertion could have had no other foundation than a conjecture. If he had told us plainly that he knew little or nothing about the matter with any degree of certainty, but that it was a conjecture of his, or the conjecture of some one else, that action and reaction are always equal to each other, no one would have been led astray; but instead of that, he declared the conjecture to be an axiom, or law of motion; and thereby his followers have been greatly deceived: for the conjecture is not more than half true.

4. In his first corollary to the Laws of Motion, Newton asserts that when two forces act upon a body at the same time, each force will impress upon the body the same velocity as that which it would have impressed if they had acted upon the body separately. Now, if Newton had known how to determine the velocities that two given forces would impress upon a body when they

both acted upon it at the same time, and also the velocities which those forces would impress when they acted upon it separately, he would have been entitled to say whether they were the same or different; but he neither knew, nor pretended to know, how to determine what velocities would be impressed in either the one case or the other; and therefore his assertion that they would be equal in the two cases, could have had no other foundation than a conjecture. If he had told us plainly that he knew little or nothing about the matter with any degree of certainty, but that it was a conjecture of his, or the conjecture of some one else, that the velocities impressed in the two cases are equal to each other, no one would have been led astray; but instead of that, he asserts, without qualification, that they are always equal; and thereby his followers have been greatly deceived: for the velocities are not, upon an average, equal in one case in a thousand.

5. In his third corollary to the Laws of Motion, Newton supposes a case in which B, weighing 3 pounds, is moving forward with a velocity of 2 feet per second, and is followed by A, weighing 1 pound, and moving with a velocity of 10 feet per second. In this case the momentum of A, before collision, is equal to 10, and the momentum of B is equal to 6. Hence the joint momentum of A and B before collision is equal to 16. And he says,—correctly,—that the joint momentum of A and B after collision will be equal to the joint momentum before collision. But he knew not how to determine what portion of A's momentum would be imparted to B at the time of collision. For he says that if B, at the time of collision, acquire 3, 4, or 5 tenths of A's momentum, then A will lose 3, 4, or 5 tenths of its own momentum; and after collision the momentum of B will be

increased to 9, 10, or 11, and the momentum of A will be reduced to 7, 6, or 5. [The Law which governs this case may be found in Section 4, of this work.] Furthermore, he says that if B, at the time of collision, acquires 9, 10, 11, or 12 tenths of A's momentum, A will lose 9, 10, 11, or 12 tenths of its own momentum; and that, after collision, B will proceed with a momentum equal to 15, 16, 17, or 18; that A will follow on after B, with a momentum equal to 1, having lost 9; or come to rest, having lost the whole of its momentum; or be reflected back from the point of collision with a momentum equal to 1, having imparted to B the whole of its own momentum and 1 tenth more; or be reflected back with a momentum equal to 2, having imparted to B the whole of its own momentum, and 2 tenths more.

From this it is evident that Newton neither knew, nor pretended to know, what portion of A's momentum would be imparted to B at the time of collision. But in order to cover the whole ground he makes three suppositions. 1. He supposes that A might impart a portion only of its momentum to B; in which case he supposed that A would follow on after B. 2. He supposes that A might impart the whole of its momentum to B; in which case he supposed that B would be brought to a state of rest. 3. He supposes that A might impart more than the whole of its momentum to B; in which case he supposed that A would be reflected back from the point of collision. Now these suppositions are all wrong. The first is wrong, for, although A would impart only a portion of its momentum to B, in the case supposed, yet A would not follow on after B, but would be reflected back from the point of collision. The second supposition is wrong because when the weight of A is either greater or

less than the weight of B, A can not impart the whole of its momentum to B in any case whatever. It can only do so when the weight of A is equal to the weight of B: and even then it can not do so if B,—as in the case supposed by Newton,—is moving forward in the same right line as that in which A is moving. The third supposition is not only wrong but absurd: for a body can not, under any circumstances, impart to another body a momentum greater than its own. This absurd notion led Newton into another absurdity. He had already said that the joint momentum of A and B, after collision, must be equal to their joint momentum before collision; which, in the case supposed, was equal to 16; 10 of which belonged to A, and 6 to B. But being possessed with the notion that A could not be reflected back from the point of collision until after it had imparted to B the whole of its momentum, and more besides, he supposed that A had imparted to B a momentum equal to 12; which made B's momentum after collision equal to 18. And as A had imparted to B the whole of its own momentum, and two parts more, he supposed that A would be reflected back with a momentum equal to those two parts. Hence the case which he had to consider was simply this: B had a momentum equal to 18, and A,—an entirely different body, and moving in an opposite direction,—had a momentum equal to 2. But 18 and 2 are equal to 20, and 20 is 4 more than 16, and 16 is all the momentum that must be found after collision. This dilemma ought to have convinced Newton that there was an error somewhere in his reckoning; but it did not; for he boldly asserts that when one body is moving in one direction with a momentum equal to 18, and another body is moving in an opposite direction with a momentum equal to 2, their joint momentum is

not equal to 20, but only equal to 16. His doctrine is, that when two bodies move in the same direction, their joint momentum is equal to the *sum* of their separate momenta; but when two bodies move in opposite directions, their joint momentum is only equal to the *difference* between their separate momenta. The absurdity of this doctrine is partly concealed by the fact that it gives him the required amount of momentum, namely, 16; for the difference between 2 and 18 is equal to 16. But if this doctrine be applied to a case,—and there are many of them,—in which the momentum of A is equal to the momentum of B, its absurdity will fully appear. Suppose the momentum of each body to be equal to 10; then their difference would be equal to nothing. Consequently their joint momentum would be equal to nothing. Hence we perceive that, in this case, Newton could not only not have found all the required momentum, but he could not have found any at all, according to the absurd method which he had invented for the very purpose of accounting for the whole of it.

Newton's doctrine amounts to the same thing as to say that if two horses move in the same direction,—say toward the east,—one at the rate of 18, and the other at the rate of 2, miles an hour, their joint rate will be equal to 20 miles an hour; but when one horse moves toward the east at the rate of 18 miles an hour, and the other toward the west at the rate of 2 miles an hour, then their joint rate is only equal to 16 miles an hour. For, although one horse actually moves at the rate of 18 miles an hour, and the other at the rate of 2 miles an hour: and although 18 and 2 are equal to 20, yet since the horses move in opposite directions, the one that moves toward the east at the rate of 18 miles an hour,

only moves at the rate of 16 miles an hour; and the one that moves toward the west at the rate of 2 miles an hour, does not move at all. Consequently their joint rate of speed is only equal to 16 miles an hour: than which there is nothing more absurd to be found in the annals of science.

The fact that Newton could not account for the reflection of a body without resorting to the absurd method above alluded to, shows that he did not understand the nature of the change which takes place when one body is reflected from another. And as no one else has ever explained the matter, it may be well, perhaps, to do so now and here.

In some cases of collision, A, at the time when it comes into collision with B at rest, is reflected back from the point of collision, and in other cases it is not reflected back, but follows on after B. And there are, also, cases in which A is neither reflected back nor follows on after B, but remains at rest at the point of collision. Now we require to know in what cases A will follow on after B; in what cases A will remain at rest; and in what cases A will be reflected back from the point of collision. According to the Laws enunciated in the preceding Sections, A will follow on after B in every case in which the weight of A is greater than the weight of B; A will remain at rest in every case in which the weight of A is equal to the weight of B; and A will be reflected back in every case in which the weight of A is less than the weight of B. Let us consider these cases separately.

In the first case,—that is, when the weight of A is greater than the weight of B,—A will not impart to B the whole of its own momentum, but only a certain portion of it; and it will then follow on after B with the re-

mainder. The amount of the momentum that will be imparted to B by A, in any given case whatever, may be determined exactly from the principle that one body can not impress upon another body, by direct action, a velocity greater than its own. For example, suppose the weight of B to be equal to 5 pounds; the weight of A 10 pounds, and the velocity of A before collision, equal to 10 feet per second. In this case the momentum of A before collision will be equal to 100. Now as A can only impart to B a velocity equal to its own, it follows that the velocity of B after collision will be equal to 10 feet per second; and this multiplied by 5,—the weight of B,—will make the momentum of B, after collision, equal to 50. And it follows that the momentum of A, after collision, will also be equal to 50; for $100-50=50$. And, having found the momentum of A after collision, we can also find the velocity of A after collision. For, the value of the number which represents the velocity of A must be such, that, when multiplied by the weight of A, the product will be equal to 50. That number, in this case, is 5. Hence A will follow on after B with a velocity equal to 5 feet per second. By the same reasoning, if B had only weighed 4 pounds, instead of 5, the momentum of B would have been 40 instead 50; and the momentum of A would have been 60 instead 50. And if B had only weighed 3 pounds, its momentum would have been 30, and the momentum of A would have been 70; and so on. And hence we are led to the Law, that the momentum of B, after collision, in every case belonging to this class,—as compared with the momentum of A before collision,—is proportional to the ratio of the weight of B to the weight of A; and also to the Law, that the momentum of A,—as compared with its own momentum before collision,—

will be proportional to the difference between the respective weights of A and B.

We have seen above that when the weight of B is made less and less, as compared with the weight of A, the momentum, and, consequently, the velocity also of A, becomes greater and greater; but an examination of the Laws will show that if the weight of B be made greater and greater, as compared with the weight of A, the velocity of A, after collision, will become less and less; and that when the weight of B becomes equal to the weight of A, the velocity of A will be equal to nothing: for then the difference between the respective weights of A and B will be equal to nothing. While the weight of B is less than the weight of A, A will impart to B a velocity equal to its own, but not a momentum equal to its own: it will always be less. But when the weight of B is equal to the weight of A, A will not only impart to B a velocity equal to its own, but also a momentum equal to its own. In other words, A will impart to B the whole of its own momentum, and then remain at rest.

When the weight of A is less than the weight of B, A never imparts to B the whole of its own momentum: it only imparts to B a portion of its momentum; and the amount thereof is determined by the relative proportion of the respective weights of A and B. For example, if the weight of A be equal to one half, or one third, or one fourth, of the weight of B; then A will impart to B one half, one third, or one fourth of its momentum. This Law is expressed by saying that the momentum of B, after collision, will be proportional to the ratio of the weight of A to the weight of B. Now, we have seen above that when the weight of A is greater than the weight of B, A imparts to B a portion of its own

momentum, and then follows on after B with the remainder. But such is not the case when the weight of A is less than the weight of B; for then A is reflected back, from the point of collision, with a momentum equal to the remainder. For example, if A imparts to B one half, one third, or one fourth, of its own momentum, then A will be reflected back with a momentum equal to one half, two thirds, or three fourths. This Law is expressed by saying that the momentum of A, after collision, will be proportional to the difference between the respective weights of A and B. But whence comes this momentum with which A is reflected back? Is it the same as that which it had before collision, or is it derived from B? It can not be derived from B, for B was at rest at the time of collision; and one body can not impart to another that which it does not possess itself. It can not be precisely the same as that which it had before collision, because that momentum was all in the opposite direction. But if we take the momentum which A had before collision, and deduct from it the amount which has been imparted to B, the remainder will be exactly equal to that with which A is reflected back from the point of collision. Hence it is evident that the momentum with which A is reflected back, consists of a portion of that which it had before collision, with its direction changed: that is, A imparts to B a portion of its momentum, and then imparts the whole of the remainder to itself in an opposite direction.

Newton supposed that A would never be reflected back until after it had imparted the whole of its momentum to B: a case which never happens when the weight of A is less than the weight of B. His theory was, that in every case in which A did not impart the whole of its momentum to B, at the time of collision, A would follow

on after B with the remainder; that when A did impart the whole of its momentum to B, A would remain at rest; and that when A imparted more than the whole of its momentum to B, A was reflected back with a momentum equal to the excess: that is, if A, having a momentum equal to 10, should impart to B a momentum equal to 12, then A would be reflected back with a momentum equal to 2. Newton does not attempt to explain how A could impart to B a momentum greater than its own; nor, supposing that to be possible, does he assert how much more momentum than its own A could impart to B; but he leaves it to be inferred that it may be any quantity whatever. And Newton not only supposed that A might impart to B a momentum greater than its own, but he also supposed that A might impart to B a greater portion of its own momentum at one time than at another; and he speaks as though the quantity imparted depended upon the velocity of A, or upon chance; and therefore could only be ascertained, in any given case, by observation and experiment. All which shows that Newton was not acquainted with the fundamental principle which makes a Law of Motion possible. There is no chance about the matter; there is no unknown cause in operation by virtue of which A may be compelled to impart a greater portion of its momentum to B at one time than at another; neither does the velocity of A determine what portion of its momentum A will impart to B; it depends wholly upon the proportion between the respective weights of A and B. For example, when the weight of A is to the weight of B, in the proportion of 1 to 3, then A,—whatever may be its velocity,—will impart to B one third of its momentum. In order to comprehend the full import and meaning of this, A must be regarded as being incapable of imparting to B either

more or less than exactly one third of its momentum; and, on the other hand, B must be regarded as incapable of receiving either more or less than exactly one third of the momentum of A. But if the weight of A is to the weight of B, in the proportion of 1 to 2, then A, whatever its velocity may be, will impart to B one half of its momentum. And if the weight of A is to the weight of B, in the proportion of 1 to 1, then A will impart to B the whole of its momentum. When the weight of A is to the weight of B, in the proportion of 2 to 1, then A will impart to B one half only of its momentum. And when the weight of A is to the weight of B in the proportion of 3 to 1, A will only impart to B one third of its momentum.

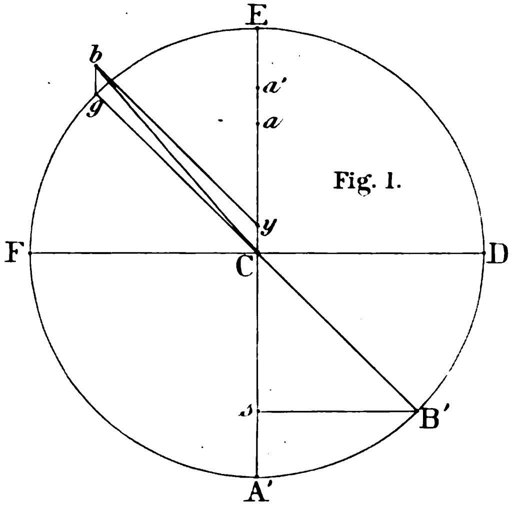
These examples include three classes of cases; namely, when the weight of A is less, equal to, and greater than, the weight of B. But as the second class can be included in either the first or the third, it is not necessary to regard them as consisting of more than two classes. The Law which governs the first class of cases has been expressed above by saying that the amount of the momentum which A will impart to B, will be proportional to the ratio of the weight of A to the weight of B. And the Law which governs the second class of cases has been expressed by saying that the amount of the momentum which A will impart to B, will be proportional to the ratio of the weight of B to the weight of A. These two Laws give different results with respect to the velocity that will be impressed upon B by A, but they give the same results with respect to the relative proportion of A's momentum that will be imparted to B by A at the time of collision: that is, when the weight of A is equal to one half of the weight of B, and when the weight of A is equal to twice the weight of B, A, in each

case, will impart to B exactly one half of its own momentum, whatever the amount of that momentum may be. And when the weight of A is three times less than the weight of B, and when the weight of A is three times greater than the weight of B, A, in each case, will impart to B exactly one third of its momentum: and so on. It also follows from these Laws that A may be substituted for B, and B for A, without changing the nature of the result. For example, suppose a case in which the weight of A is equal to 1 pound; the weight of B equal to 3 pounds; and in which B is at rest at the time of collision. In this case A will impart to B, at the time of collision, one third of the momentum which it had before collision. If, now, we suppose A to be at rest, and B the moving body, B, at the time of collision, will also impart to A one third of the momentum which it had before collision. Hence, when it is known what portion of its momentum A, in any given case, will impart to B at rest, it also becomes known what portion of its momentum B would impart to A at rest, in that case.

In like manner a reason can be given for every single Law, and for every set of Laws, thus far enunciated in this work. But as those reasons become more numerous and more complicated as we advance, it would be unreasonable to suppose that any reader would follow me into such a labyrinth. Hence no further attempt will be made to show that those Laws are true by reasoning from principles. Besides there is no necessity for so doing. Those Laws could not have been discovered without a previous knowledge of all the principles involved; but, after being discovered, their truth may be established without the aid of any principle whatever; for those Laws are of such a nature that, if they

are false, they will contradict themselves by giving, in some cases, either more or less momentum after than before collision; but if they are true they will give the same momentum after as before collision, in all cases whatsoever. Hence, if any one will find a case in which the Laws do not hold good, he will establish, beyond all peradventure, that the particular set of Laws to which the case belongs is false. But it is here fearlessly asserted that those Laws are not only the true Laws, but they are also the only Laws that are possible. Any other supposed Laws will be found to involve a mathematical self-contradiction.

Plate, I.



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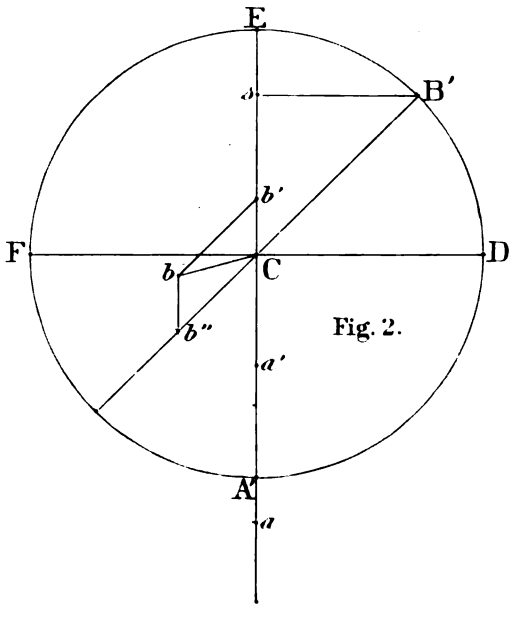


Fig. 2.

Plate. 2.

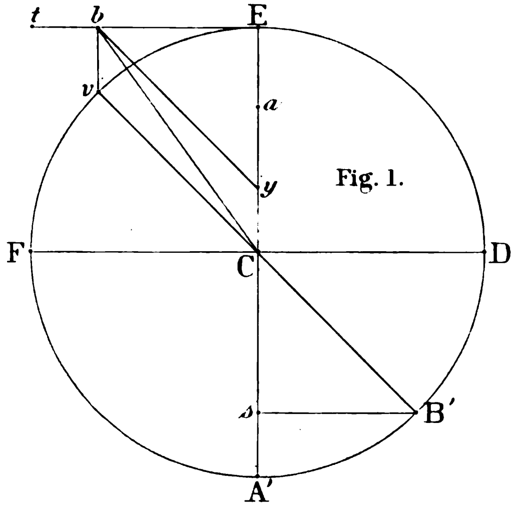


Fig. 1.

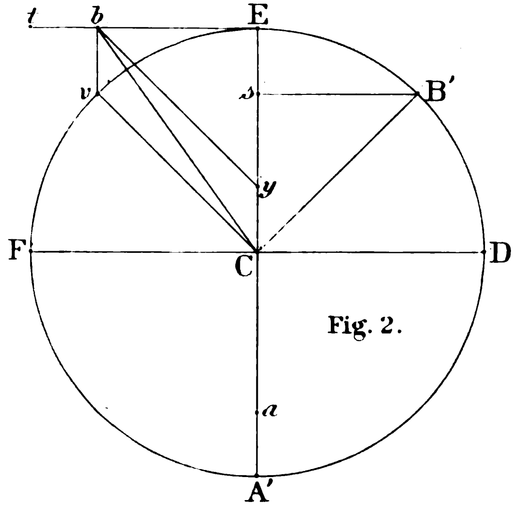
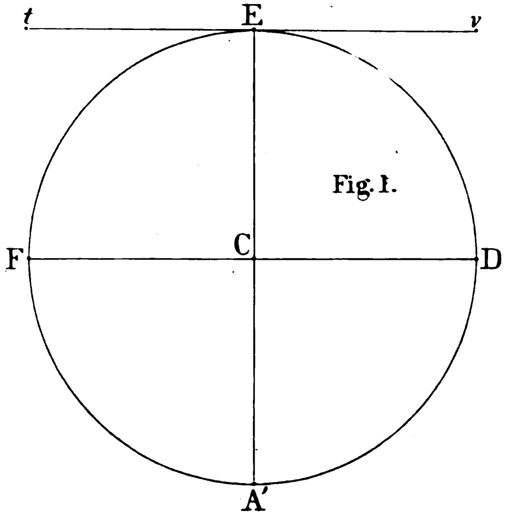


Fig. 2.

Plate. 3.



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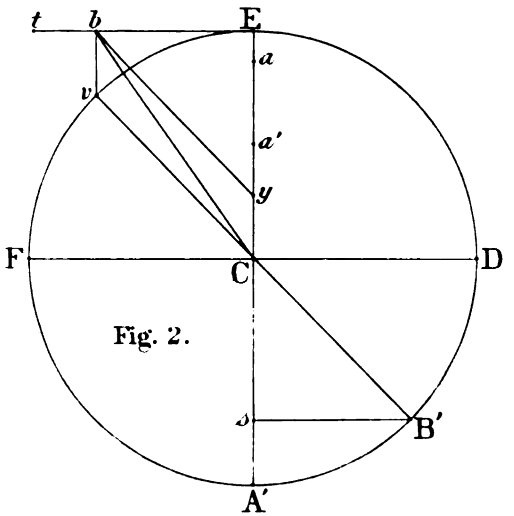
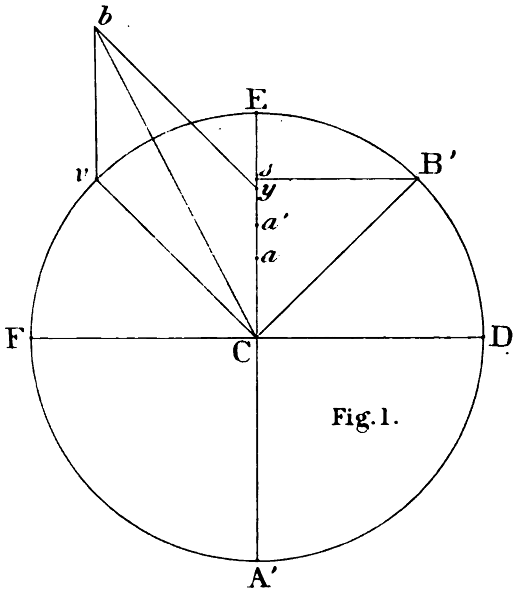
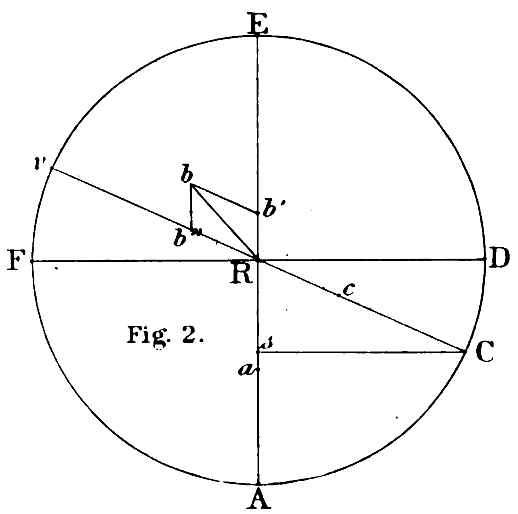


Fig. 2.

Plate. 4.





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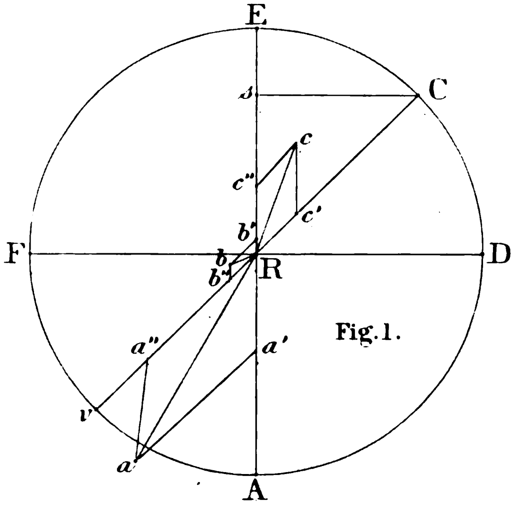

Fig. 1.

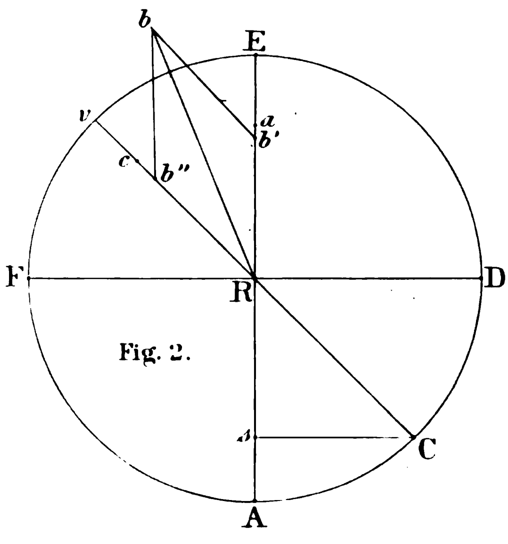

Fig. 2.

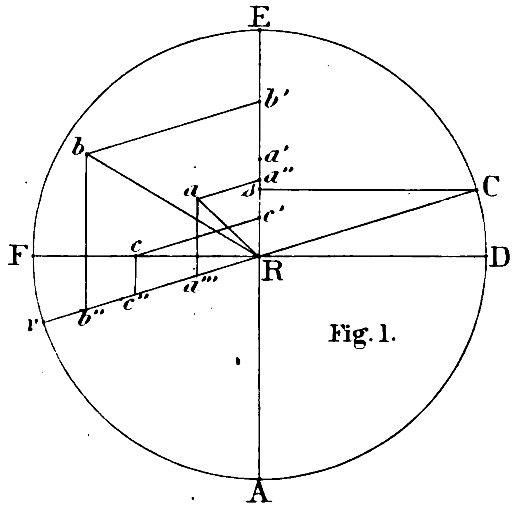
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Fig. 1.
A

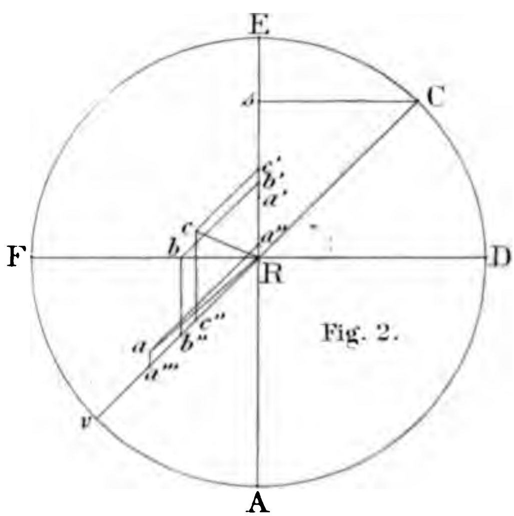
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Fig. 2.
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